Investment Dynamics in Electricity Markets*

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Abstract. We introduce a simple strategic dynamic model with random demand growth to investigate the incentives for capacity investments. We study Markovian equilibria where the firms' decisions depend on the current capacity stock only. For these non-collusive equilibria, unexpected investments allow the firms to increase their market share, but they also generate excess capacity that drastically depresses the spot price. The second effect is stronger and the firms maintain very little excess capacity. In some equilibria, even rationing occurs with positive probability along the equilibrium path. Thus, the market does not provide adequate incentives to ensure 'security of supply'. Moreover, the tight capacity constraint allows the firms to maintain very high market prices and extract huge rents. Traditionally, the regulator sets the price cap equal to the social cost of a marginal rationing. Our welfare analysis suggest that this value is too high and a different calibration is required to reduce market prices and increase consumer surplus, without affecting the level of investment.

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1. Introduction

The new electricity markets that have emerged around the world since the early 1990s, are often characterized by an oligopoly of generators, almost no short term price elasticity of demand, and complex, administered market mechanisms which are designed to facilitate both financial trading and physical, real-time system balancing (see for instance Stoft (2002) and Wilson (2002)).

In the short run, the reliability of electricity is typically measured by the frequency and duration of interruptions that are associated with small scale and localized imbalances in generation and transmission of capacity and demand. In the long run, however, reliability is associated with the slack of installed generation and transmission capacity over the demand forecast. Thus, long-run reliability depends on the likelihood of imbalances in new capacity investment and future demand growth.

In most restructured power systems, reliability in the short-run is the responsibility of the system operator (ISO) while reliability in the long run is left to market forces. It is frequently argued that the market has a self regulating ability to provide the socially optimal level of reliability in the long run. In this paper, we study the dynamics of capacity investments in a strategic model of electricity markets without considerations of transmission constraints. Thus, our model provides a framework to analyze the provision of "security of supply".

The model presents a simplified but realistic electricity market. Every period, capacity-constrained firms bid in the spot market for the right to generate and sell electricity, and then invest in new capacity. Demand grows randomly. When there is little excess capacity, the price competition in the spot market produces high prices and generous rents. Since a firm's rents are proportional to its market share, the firms have strong incentives to invest and maintain or increase their market share. But, large investments generate excess capacity and relatively low spot prices and rents. A question we investigate here is whether the market provides appropriate incentives for the firms to generate an adequate level of investment. This question is related to the public policy debate that followed the California debacle, where it was frequently argued that "resource adequacy" or "security of supply" would be guaranteed by simply relaxing the price cap restriction.

The thrust of our results is mostly negative. In equilibrium, the firms tend to maintain a very tight capacity constraint and, in some cases, capacity is often insufficient to cover demand and rationing occurs. In principle, maintaining excess capacity is wasteful, and one may view this this feature of equilibrium as a positive result. However, in addition to the social cost of rationing, the lack of excess capacity means that the firms have little incentive to compete in the bidding process. Consequently, the firms are able to sustain very high market prices and extract big surpluses. Our model has multiplicity of equilibria and it would be easy to construct 'collusive' equilibria, where the firms use intertemporal incentives to obtain these results. For this reason, we restrict attention to Markovia equilibria that preclude this type of incentives. Depending on the current capacity stock, the spot market auction also sometimes admits a multiplicity of equilibria. Here, again, we select the 'most competitive' equilibrium, in the sense that it produces the lowest (expected) spot price.

We construct three equilibria that share many features, but are adapted to cover different ranges for the parameters of the model: risk free interest rate, price cap, demand growth rate, and investment cost. In equilibrium, the firms never invest more than what is required to cover a possible demand growth. Therefore, excess capacity never exceeds the size of the largest demand jump. In fact, in our second equilibrium, the firms invest even less and rationing occurs with positive probability along the equilibrium path. Since demand jumps are relatively small, the firms are guaranteed to be dispatched for a substantial portion of their installed capacity. Consequently, the firms do not compete very aggressively in the spot market auction and spot prices are very high. In our first equilibrium, the firms invest exactly to offset the largest demand jump. Therefore, there is security of supply, and there is excess capacity in those periods in which demand grows less than the maximal demand jump. In the second equilibrium, the firms only invest to match *current* demand. So there is rationing in periods when demand grows. The firms are better off in this equilibrium when the probability of demand growth is low. Interestingly, above a certain threshold, increasing the price cap does not change these equilibria. That is, increasing the price cap has no effect on the level of investment. Our model assumes only two firms. We have not studied how the equilibrium changes as the number of firms increases, and therefore we do not know whether relaxing the price cap helps when there is free entry.

In the model with two firms, *decreasing* the price cap may have a positive impact on welfare without affecting the level of investment. The benefit here is similar to the benefit of imposing a price cap on a monopolist. As in the standard textbook example, to maximize consumer surplus, we would like to lower the price cap to the point where the firms make just enough profits to pay for the investment costs.

The range of parameters for which the first two strategies are equilibria is severely reduced as the probability of demand growth increases to 1. In the third equilibrium, we concentrate in the case where the probability of a demand increase is 1 and therefore returns on investment are relatively large. Here, the temptation to grab market share is strong. When the current total capacity exceeds demand, our equilibrium requires that the firms make large investments and equalize their capacities. After that initial investment, the firms make small investments and slowly allow demand to catch up with total installed capacity. Along this trajectory, the spot price remains relatively low and the firms collect small rents. This discourages the firms from trying to grab market share when the total capacity is just enough to cover demand. Any unexpected increase in capacity leads to the unattractive trajectory.

Most of the literature on electricity markets has focused on strategic behavior in the short-run (see for instance Green and Newberry (1992), Von der Fehr and Harbord (1993), Borenstein and Bushnell (1999) and Wolfram (1998) among others). The subtle effects of congestion have also been studied (see Borenstein et al. (2000) and Joskow and Tirole (2001)). However, the dynamics of investment decisions have received less attention. This paper is a contribution towards a clearer understanding of the provision of "security of supply" and, more broadly, of the determinants of social welfare in electricity markets.

Section 2 presents a duopoly model of an electricity market with random demand growth and market rules similar to those commonly in place. Section 3 analyzes the price auction game that the firms play in each bidding cycle. Because we restrict attention to a particular class of Markovian equilibria, the strategic problem of the firms in every bidding cycle is independent of the rest of the game. Section 4 exploits the homogeneity of the payoff functions in the auction games to construct very simple Markovian equilibria. In these equilibria, the investments decisions are also homogeneous of degree 1 in the current capital stocks, and are independent of the history of the game, including the outcome of the last auction. Their simplicity simplifies the analysis enormously. In Section 5 we construct three equilibria. One of our equilibria has insufficient capacity investment along the equilibrium path and shortages occur with positive probability. In all three, the firms manage to sustain very high market prices and extract huge surpluses. Section 6 contains our conclusions.

2. Model

In this section we introduce a simplified dynamic model of strategic investments in electricity markets. In each period, the firms participate in a uniform price auction that specifies the market price and the fraction of each firm's capacity that is utilized. After the firms realize their profits for the current period, they invest in new capacity. New capacity becomes available immediately the next period; old capacity does not depreciate. Demand grows stochastically over time.

Assume there are 2 firms. Each firm has a constant marginal cost of production $c > 0$ up to its current capacity. A price cap $\bar{p} > c$ is stipulated by the regulatory commission. Denote by $m = \bar{p} - c$ the maximum markup allowed by the commission. Let $K^t = (K_1^t, K_2^t)$ be the firms' capacities and D_t be the inelastic demand in period t.

Firm i has n_i plants (units). For simplicity (and to keep the state space of the game to a minimum size), we assume that for all t, the total capacity K_i^t of firm i is equally divided among its plants, so each plant has size $s_i^t = K_i^t / n_i$.

In the price auction, firm i submits n_i bids $b^i = (b_1^i, ..., b_{n_i}^i)$, where $0 \leq b_1^i \leq \cdots \leq b_{n_i}^i \leq \bar{p}$. All the plants are ordered from lowest to highest bid, with ties broken in favor of firm 1 (if $b_k^1 = b_\ell^2$, then the k-th plant of firm 1 is listed ahead the ℓ-th plant of firm 2), and then they are dispatched in that order until their combined capacity is greater than or equal to D_t. If the total capacity is less than or equal to demand, that is, if $K_t^1 + K_t^2 \leq D_t$, there is no marginal plant and the spot price is set equal to $\bar{p}$. Otherwise the spot price is set equal to the bid of the *marginal plant*. [1] Assume that only the first k plants are dispatched. The marginal plant is the k-th plant (the last dispatched plant) if the combined capacity of the first k plants is strictly more than D_t, or the $(k+1)$-th plant otherwise. That is, the marginal plant is that plant that would be required to cover demand if D_t were to increase by one unit. If the last dispatched plant is the marginal plant, then it is only dispatched for the capacity required to cover demand. Let p_t be the spot price and q_i^t be the total capacity demanded from firm i as a result of the auction. The net revenue of firm i in period t is $R_i^t = (p_t - c) q_i^t$.

At the end of each period t, the firms simultaneously choose capacity investments $Y_i^t \geq 0$, $i = 1, 2$. The constant marginal cost of investment is $\kappa > 0$. Hence, firm i's net profit

[1] See Von der Fehr et al. (2002) for a discussion on the properties of this auction format versus a discriminatory auction format in which dispatched plants are paid according to their bids. A brief overview of the different auction formats adopted in several countries is also given.

for period t is $\pi_i^t = R_i^t - \kappa Y_i^t$, and its capacity for next period becomes $K_i^{t+1} = K_i^t + Y_i^t$. Demand grows randomly: for all $t \geq 0$, $D_{t+1} = (1+g)D_t$ with probability θ and $D_{t+1} = D_t$ with probability $1 - \theta$. The growth rate $g > 0$ is constant over time. Firm i's total discounted (and normalized) payoff is

$$(1-\beta)\sum_{t\geq 0} \beta^t \pi_i^t,$$

where $\beta \in (0,1)$ is the discount factor (assumed the same for both firms).

This is a dynamic game that for sufficiently high discount factor β has many (subgame perfect) equilibria. We restrict attention to a symmetric Markovian equilibrium where the bidding strategies and the investment strategies of the firms depend only on the current state (D_t, K^t). Though in the component game of period t each firm moves twice (the first time to choose a bid and the second to choose an investment), in our Markovian equilibrium the investment decision is independent of the outcome of the price auction. This Markovian equilibrium is extremely simple because it treats the price auction of each period as an independent game. In general, there are collusive equilibria where relatively unaggressive bidding (high prices) is supported by the promise of higher continuation values. In our Markovian equilibrium there are no intertemporal incentives for the price auction game, and therefore it must prescribe an equilibrium for the price auction game of each period. We next study the price auction game in isolation. Once we construct an equilibrium for this game, we study the investment decision problems of the firms.

The relevant literature on dynamic investment, features the works by Spence (1979) and Fudenberg and Tirole (1983). In the latter, firms build capacity smoothly at bounded rates over time. It is shown that in addition to the equilibrium identified by Spence (1979) (in which firms accumulate capacity to reach the Cournot equilibrium) there are equilibria in which the firms maintain less capacity (e.g. they can even attain the monopoly total capacity and split the profits). Besanko and Doraszelski (2004) have also used a similar setup to study, via numerical experiments, the relation between the nature of short-run competition and long-run asymmetries in firm size.

3. The Price Auction Game

Assume that the current capacities are $K = (K_1, K_2)$ and that current demand is D. We need to consider 5 separate cases. For most of the analysis, we consider a restricted class of *symmetric strategies profiles* b, where $b_1^i = b_2^i = \cdots = b_{n_i}^i = p_i$, $i = 1, 2$. In a symmetric strategy, firm i bids a common price p_i for all its units. To simplify, we denote a symmetric strategy profile by (p_1, p_2).

Case 1: $K_1 + K_2 \leq D$. Here there is a unique equilibrium outcome. The market price is $\bar{p}$, both firms are dispatched up to their capacities (there is rationing) and the corresponding revenues are $R^* = (mK_1, mK_2)$. Though any bidding strategy is an equilibrium, we select in this case the symmetric strategy profile $(\bar{p}, \bar{p})$ where firm 1 and firm 2 bid all their plants at the common price $\overline{p}$.

Case 2: $K_i < D$, $i = 1, 2$, and $K_1 + K_2 > D$. There are two symmetric pure strategy equilibria and a continuum of symmetric mixed strategy equilibria. The pure strategy equilibria are $(p_1, p_2) = (c, \overline{p})$ and $(p_1, p_2) = (\overline{p}, c)$, with corresponding revenues $R = (mK_1, m(D - K_1))$ and $R = (m(D - K_2), mK_2)$ respectively. In any symmetric mixed-strategy equilibrium, each firm i chooses p_i randomly in the interval $[c, \overline{p}]$ according to a distribution Φ_i. It is easy to argue that in the interval $[c, \overline{p})$, Φ_i is absolutely continuous with a density φ_i. However, it is possible that either Φ_1 or Φ_2, but not both, has a jump at $\bar{p}$. Let $\bar{\varphi}_i = \Phi_i(\bar{p}) - \lim_{p\uparrow\bar{p}} \Phi_i(p) = 1 - \lim_{p\uparrow\bar{p}} \Phi_i(p)$. Then $\bar{\varphi}_i \geq 0$, $i = 1, 2$, and $\bar{\varphi}_1\bar{\varphi}_2 = 0$. The expected revenue for firm 1 when it bids a price $p_1 \in [c, \overline{p})$ is

$$K_1 \left[\int_{p_1}^{\overline{p}} (p_2 - c)\varphi_2(p_2)dp_2 + (\bar{p} - c)\bar{\varphi}_2 \right] + (D - K_2)(p_1 - c)\Phi_2(p_1).$$

Since firm 1 is randomizing, it must be that this expected revenue does not depend on p_1, and therefore the derivative of the above expression with respect to p_1 must be 0 for all $p_1 \in (c, \overline{p})$:

$$-K_1(p_1 - c)\varphi_2(p_1) + (D - K_2)\Phi_2(p_1) + (D - K_2)(p_1 - c)\varphi_2(p_1) = 0.$$

Simplifying, we obtain $\varphi_2(p) = A\Phi_2(p)/[p - c]$, where $A = [D - K_2]/[K_1 + K_2 - D]$. The solution of this differential equation[2] with boundary condition $\lim_{p\uparrow\bar{p}} \Phi_2(p) = 1 - \bar{\varphi}_2$ is

$$\Phi_2(p) = (1 - \bar{\varphi}_2) \left[\frac{p - c}{m} \right]^A \quad \text{and} \quad \varphi_2(p) = (1 - \bar{\varphi}_2)\frac{A}{m^A}[p - c]^{A-1} \quad \text{for all } p \in [c, \bar{p}).$$

The strategy for firm 1 is symmetrically constructed. The corresponding revenues are $R = (m[(1 - \bar{\varphi}_2)(D - K_2) + \bar{\varphi}_2 K_1], m[(1 - \bar{\varphi}_1)(D - K_1) + \bar{\varphi}_1 K_2])$. The probabilities $(\bar{\varphi}_1, \bar{\varphi}_1)$ satisfy $\bar{\varphi}_i \in [0, 1)$, $i = 1, 2$, and $\bar{\varphi}_1\bar{\varphi}_2 = 0$. Since $K_1 + K_2 > D$, the 'most competitive' equilibrium, that is the equilibrium with the lowest revenues for the firms, corresponds to the case $\bar{\varphi}_1 = \bar{\varphi}_2 = 0$ with $R^* = (m(D - K_2), m(D - K_1))$. Note that with this choice, the equilibrium simultaneously delivers the minmax net revenue for each firm and is also more competitive than the two pure strategy equilibria above. In the Markovian equilibrium we study, we select this bidding equilibrium.

Given the other firm plays its symmetric mixed strategy, our derivation above makes sure that firm i is indifferent about which common price to bid for all its units. However, we also need to check that other asymmetric strategies are not more profitable. To simplify the notation, let us consider just an example. Suppose that $D = 5.5$, $K_1 = K_2 = 4$, and that firm 1's bid b^1 translates into capacities of 1 being bid at price q_ℓ, $\ell = 1, \ldots, 4$, where $c \leq q_1 < q_2 < q_3 < q_4 < \bar{p}$. Firm 1's expected net revenue for this strategy is

$$1.5(q_2-c)\Phi_2(q_2)+2\int_{q_2}^{q_3}(p_2-c)\varphi_2(p_2)dp_2+3\int_{q_3}^{q_4}(p_2-c)\varphi_2(p_2)dp_2+4\int_{q_4}^{\bar{p}}(p_2-c)\varphi_2(p_2)dp_2.$$

Note that whether $p_2 \in [c, q_1)$ or $p_2 \in [q_1, q_2)$, firm 1 dispatches a total capacity of 1.5 and the spot price is q_2. So firm 1 might as well choose to bid a capacity of 2 at the common price q_2. Clearly, this expected revenue is smaller than

$$1.5(q_2 - c)\Phi_2(q_2) + 4\int_{q_2}^{\bar{p}}(p_2 - c)\varphi_2(p_2)dp_2,$$

[2] See Fabra et al. (2003) for a more complete derivation.

which firm 1 could attain if instead it bid all its capacity at the common price q_2. Therefore, there are no profitable asymmetric deviations. Though the example is particular, the argument is clearly general and would apply to any D, $(K_1, K_2) \in (0, D)^2$ with $K_1 + K_2 > D$, and any b^1.

In the selected equilibrium, though a firm's net revenue depends on its opponents' capacity only, the expected price markup (and hence consumer surplus) depends on the joint excess capacity. Let $E = K_1 + K_2 - D$ denote excess capacity and $a = D/E - 1$. By assumption, $0 < E \leq D$. Then, $\text{Prob}[p_t \leq p] = \Phi_1(p)\Phi_2(p) = [(p-c)/m]^a$. Therefore, the expected price in the mixed-strategy equilibrium is

$$\hat{p} = \int_c^{\bar{p}} pa \frac{(p-c)^{a-1}}{m^a} dp = (\overline{p} - c) \left[1 - \frac{E}{D}\right] + c.$$

As $E \to 0$, $\hat{p} \to \bar{p}$, and when $E = D$, $\hat{p} = c$.

Cases 3–4: $K_i \geq D$ and $K_j \leq D$. When D is an integer multiple of the plant capacity $s_i = K_i/n_i$, that is when $D = \ell^* s_i$ for some positive integer ℓ^*, there is a "fragile" asymmetric equilibrium that gives all the revenue to firm i. In that equilibrium, $b^i_\ell = c$ for $\ell \leq \ell^*$ and $b^i_\ell = \bar{p}$ for $\ell > \ell^*$, while $b^j_\ell = \bar{p}$ for all ℓ. This is an equilibrium because of the definition of the spot price. In this equilibrium, the marginal plant is not being dispatched, but it determines the spot price. When D is not an integer multiple of s_i, this equilibrium does not exist. Of course, since firm i chooses its investments, it can do so in such a way that D is always an integer multiple of s_i. This is possible for the simple model of demand growth we adopted, but would not be possible for a more realistic model. If demand growth were a continuum random variable (with mean gD), for example, then firm i would never be able to predict D exactly. Therefore, we will never consider this equilibrium. There is also a continuum of symmetric equilibria: $(p_i, p_j) = (\bar{p}, p_j)$ with $p_j \in [c, c+m(D-K_j)/D]$, all leading to the same revenues $(R^*_i, R^*_j) = (m(D - K_j), mK_j)$. For simplicity, below we will choose the symmetric equilibrium $(p_i, p_j) = (\bar{p}, c)$. When $K_j < D$, these are the only pure strategy equilibria,[3] but when $K_j = D$, for any $p_i \in [c, \bar{p}]$, (p_i, c) is also an equilibrium. When $K_j = D$ and $K_i > D$, we select again the equilibrium $(p_i, p_j) = (\bar{p}, c)$ with net revenues $(R^*_i, R^*_j) = (0, mD)$. This is not the most competitive equilibrium; here (c, c) is also a symmetric equilibrium with net revenues $R = (0, 0)$. However, as we explain at the end of Section 4, this choice is required to ensure that the net revenue function R^* has the appropriate continuity 'from below'. When $K = (D, D)$, there is a continuum of symmetric equilibria: $p = (c, p_2)$ and $p = (p_1, c)$ are equilibria for all $p_1, p_2 \in [c, \bar{p}]$. In this case, in the dynamic investment game (described in the next section), we need to select one of the three equilibria $p = (c, \bar{p})$, $p = (\bar{p}, c)$, or (c, c), depending on the investments made by the firms in the previous period.[4]

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[3] There are other (asymmetric) mixed strategy equilibria, where firm j chooses randomly for each of its units a price $b^j_k \in [c, c + m(D - K_j)/D]$, but they all have the same net revenues $(R^*_i, R^*_j) = (m(D - K_j), mK_j)$.

[4] Strictly speaking, our Markovian equilibria require a larger state vector that includes (D_{t-1}, K^{t-1}) in addition to (D_t, K^t). However, remembering (D_{t-1}, K^{t-1}) is only required when $K^t = (D_t, D_t)$ and in our analysis we are able to deal with this case by using the relevant continuation values.

Case 5: $K_i \geq D$ for $i = 1, 2$. In this case, each firm can cover the whole demand with its own capacity, and the standard Bertrand outcome attains, $(p_1, p_2) = (c, c)$, and the firms make no profits: $R^* = (0, 0)$.

To summarize, partition the capacity space $\mathbf{R}^2_+$ minus the point (D, D) into 5 regions:

$$
\begin{aligned}
S_1 &= \{K \mid K_1 + K_2 \leq D, \text{ and } K_i \geq 0 \text{ for } i = 1, 2\},\\
S_2 &= \{K \mid K_1 + K_2 > D, \text{ and } K_i < D \text{ for } i = 1, 2\},\\
S_3 &= [D, \infty) \times [0, D] \backslash \{(D, D)\},\\
S_4 &= [0, D] \times [D, \infty) \backslash \{(D, D)\},\\
S_5 &= (D, \infty) \times (D, \infty).
\end{aligned}
$$

Then, the revenue function for the players and corresponding equilibrium strategy we have selected are

$$
R^*(K, D) = \begin{cases}
(mK_1, mK_2) & \text{with } (\overline{p}, \overline{p}), & \text{if } K \in S_1\\
(m(D - K_2), m(D - K_1)) & \text{with } (\varphi_1, \varphi_2), & \text{if } K \in S_2\\
(m(D - K_2), mK_2) & \text{with } (\overline{p}, c), & \text{if } K \in S_3\\
(mK_1, m(D - K_1)) & \text{with } (c, \overline{p}), & \text{if } K \in S_4\\
(mD, 0),\ (0, mD) \text{ or } (0, 0) & \text{with } (c, \overline{p}),\ (\overline{p}, c), \text{ or } (c, c) & \text{if } K = (D, D)\\
(0, 0) & \text{with } (c, c), & \text{if } K \in S_5.
\end{cases}
$$

4. The Dynamic Investment Game

We now assume that the firms' behavior at the auction games is fixed at the bidding equilibrium strategies selected in the previous section. Fixing the behavior of the firms at the auctions produces a residual dynamic game where the firms only choose investments. Note that the equilibrium revenue function $R^*(K, D)$ is homogeneous of degree 1; let $r^*(K) = R^*(K, 1)$, so $R^*(K, D) = D \cdot r^*(K/D)$. We restrict attention to investment strategies where the decisions of the firms in period t depend exclusively on the current capital stock K^t and demand D_t. Let $Y(K^t, D_t) = (Y_1(K^t, D_t), Y_2(K^t, D_t))$ denote the profile of capacity investments in period t. Moreover, to transform the dynamic game into a stationary game, we will also require that $Y(K^t, D_t)$ be homogeneous of degree 1. Let $y(K) = Y(K, 1)$ denote the investment when the current demand is 1. Then, we assume that $Y(K, D) = D \cdot y(K/D)$.

Starting from K^0, let $\{K^t\}_{t \geq 0}$ be the (stochastic) sequence of capacity stocks when the firms follow the strategy Y. That is, for each $t \geq 0$, $K_i^{t+1} = K_i^t + Y_i(K^t, D_t)$. Define the "detrended" capacity stock (stochastic) sequence $\{k^t\}_{t \geq 0}$ by $k_i^t = K_i^t / D_t$ for all i and $t \geq 0$. Then, for each i and $t \geq 0$, $k_i^{t+1} = [k_i^t + y_i(k^t)]/[1 + g]$ with probability θ, and $k_i^{t+1} = k_i^t + y_i(k^t)$ with probability $1 - \theta$.

Let $V(K|Y, D_0)$ denote the total expected discounted payoff of the firms when the initial capital stock is $K^0 = K$, given that the firms follow the investment strategy Y and that the demand in the initial period is D_0. Observe that by homogeneity, $V(K|Y, D_0) = D_0 V(K/D_0|Y, 1)$, and without loss of generality it is enough to study the case when initial demand is $D_0 = 1$.

Let $v_i(k|y) = V_i(k|Y, 1)$, $i = 1, 2$, and note that

$$v_i(k|y) = [r_i^*(k) - \kappa y_i(k)] + \beta \left[\theta(1+g) v_i \left(\frac{k + y(k)}{1+g} \;\middle|\; y \right) + (1-\theta) v_i(k + y(k)|y) \right].$$

This identity suggests an interpretation of our model in terms of a "stationary" model. Formally, for homogeneous investment strategies, our model is equivalent to a model with a stationary demand of 1 and random discount rate and capital depreciation. Let δ be such $1 - \delta = [1+g]^{-1}$. Then, the (discount rate, capital depreciation) pair is $(\beta(1+g), \delta)$, with probability θ and $(\beta, 0)$ with probability $1 - \theta$.

Definition: Let $\gamma = \beta(1 + \theta g)$ denote the expected discount rate. We shall assume $\gamma < 1$ or equivalently, $\theta g < \rho$. Also, let ρ be the interest rate, so that $\beta = [1 + \rho]^{-1}$.

In the analysis that follows, we find it easier to work with the stationary model. The following definition implicitly uses the principle of unimprovability.

Definition: An investment function y^* is a subgame perfect equilibrium of the stationary investment game if for all k, i and $\hat{y} = (x_i, y^*_{-i}(k))$ with $x_i \geq 0$,

$$v_i(k|y^*) \geq [r_i^*(k) - \kappa x_i] + \beta \left[\theta(1+g) v_i \left(\frac{k + \hat{y}}{1+g} \;\middle|\; y^* \right) + (1-\theta) v_i(k + \hat{y}|y^*) \right].$$

One can easily check that: (1) If y^* is a subgame perfect equilibrium of the stationary investment game, then the corresponding homogeneous strategy Y^* is a subgame perfect equilibrium of the investment game (with stochastic demand growth). (2) The full strategy obtained by combining an equilibrium strategy Y^* of the dynamic investment game and the equilibrium strategies of the auction games (specified in Section 3) constitutes a subgame perfect equilibrium of the full dynamic game.

An equilibrium strategy Y^* constructed from an equilibrium strategy y^* of the stationary game is by definition homogeneous of degree 1. However, the investment game may also have non-homogeneous equilibrium strategies. Though homogeneous equilibrium strategies are intuitively appealing, our focus on homogeneous equilibrium strategies is motivated by their simplicity.

Remark: We can now explain our choice of bidding strategies (and consequently, our definition of $R^*(K, 1)$) when $K_1 = 1$ and $K_2 > 1$ (or when $K_1 > 1$ and $K_2 = 1$). As we discussed in Section 3, there are multiple bidding equilibria in this case. However, to guarantee existence of Markovian equilibrium in the dynamic game, it is necessary to select the symmetric bidding equilibrium $(c, \bar{p})$. We now argue this point informally. Assume for simplicity that $\theta = 1$, so demand grows by $(1 + g)$ every period. The equilibrium revenue function $r^*(k)$ is discontinuous at any $k = (1, k_2)$ with $k_2 > 1$. This generates a discontinuous objective function for the firm's investment problem. Consider a situation where $k_2^0 > 1 + g$ and $k_1^0 < 1 + g$. Here, even if firm 2 makes no investment, $k_2^1 = k_2^0/(1+g) > 1$ and firm 1 can induce revenues $r^*(k^1)$ in period 1 arbitrarily close to $(m, 0)$ by choosing an investment so that k_1^1 is just below 1. (Recall that when $k_1^1 < 1 < k_2^1$, there is a *unique* bidding equilibrium with revenues $(mk_1^1, m(1 - k_1^1))$.) When we set

$r^*(1, k_2^1) = (m, 0)$, as we did in the definition of the revenue function, firm 1 can optimally choose $k_1^1 = 1$. But, suppose for a moment that $r^*(1, k_2^1) = (0, 0)$, as it would be if the symmetric bidding equilibrium at $(1, k_2^1)$ were (c, c) instead, and assume that $v_1(k^2|y^*)$ is continuous in k^2 (in a neighborhood of the relevant k^2). Then, firm 1's investment decision problem in period 0 would have an objective function that is not left-continuous, and hence has no solution (firm 1 would like to maximize its investment subject to $k_1^1 < 1$). Also, when $k^{t+1} = (1, 1)$, what equilibrium and revenues are selected should depend on the investments made in period t. If, for example, firm 1 made a positive investment while firm 2 made no investment, then we need to set $r^*(k^{t+1}) = (m, 0)$ because by decreasing its investment in period t, firm 1 can guarantee revenues arbitrarily close to $(m, 0)$ in period $t+1$. On the other hand, if $k^{t+1} = (1, 1)$ and both firms made positive investments the previous period, then we set $r^*(k^{t+1}) = (0, 0)$. Thus, we need to enlarge the state space for our Markovian strategies, and when the capital stock visits the point $(1, 1)$, we need to recall what investments were made in the previous period. But, as long as we make r^* left-continuous at the boundary $\{1\} \times (1, \infty) \cup (0, 1) \times \{1\}$, we do not need to recall the last investment for any other stock. To keep the definition of the Markovian strategy as simple as possible, we prefer to allow the use of memory only for the state $(1, 1)$ (where it is unavoidable). Also, letting the strategy depend on the previous investments and the current capacity stock everywhere, allows the design of some collusive strategies that we would like to exclude.

5. Investment Equilibrium

In this section we refer exclusively to the stationary game. We are interested in two types of investment equilibria, one where total capacity is at leat equal to demand in every period with probability 1, and another where with positive probability capacity is insufficient and there is rationing.

In our equilibria, if initially there is no excess capacity, the firms keep the capacity stock in the region S_1 all the time. In that region, a firm's revenue increases with its market share. Therefore the firms would like to increase their market shares. But, excess capacity has a negative impact on the spot price, and thus investing too much in an attempt to grab market share is not immediately rewarding. Moreover, when an overinvestment sends the capacity stock to the region S_2, a firm's revenue does not depend on its own capacity. Nevertheless, a firm may expect to profit from an unexpected overinvestment in future periods, when excess capacity disappears with demand growth, if the opponent does not react and allows the firm to increase its market share. In the Markovian equilibrium of Theorem 1 below, for example, the opponent does not react: the firms make no investments in region A (that corresponds to a main portion of S_2) and allow demand to catch up with capacity. However, market grabbing is dampened by this process and in the end it is too costly to increase the market share. For similar reasons, it is not easy to construct (Markovian) equilibria that maintain excess capacity over time. Nevertheless, in Theorem 3 below we present an equilibrium with this property. If initially there is excess capacity, that equilibrium generates a symmetric trajectory for the capacity stock that has excess capacity in *every* period.

5.1 *An Equilibrium with Security of Supply*

We first present an investment strategy that in every period produces an aggregate investment equal to the size of a potential demand growth. Since investment decisions are made before the realization of demand growth, along the outcome path of this equilibrium, there will be periods (when demand does not grow) where total capacity strictly exceeds demand. Thus, this strategy ensures that demand is always served and keeps overcapacity to a minimum. We identify conditions for this strategy to be an equilibrium. As we shall see below, this is indeed the case for a wide range of parameters, including cases in which the probability of demand growth is relatively small. When that probability is small, investments are likely to produce overcapacity and therefore they do not seem attractive. However, a firm that does not invest, as required by the strategy, loses market share and therefore concedes to its opponent future rents that it could have collected itself.

Let $\mathbf{N} = \{0, 1, \dots\}$. Define the regions

$$U = \{k \geq 0 \mid 1 > k_2 > k_1/g \text{ and } k_2 > k_1 + (1-g)/(1+g)\},$$
$$W = \{k \geq 0 \mid 1 > k_1 > k_2/g \text{ and } k_1 > k_2 + (1-g)/(1+g)\},$$
$$L = \{(1,1) > k \geq 0 \mid k_1 + k_2 < 1 + g\} \backslash \{U \cup W\},$$
$$A = \{(1,1) > k \mid k_1 + k_2 \geq 1 + g\},$$
$$I_r = [0, 1+g] \times [(1+g)^r, (1+g)^{r+1}] \quad \text{for } r \in \mathbf{N},$$

and $I = \cup_{r \geq 0} I_r = [0, 1+g] \times [1, \infty)$ (see Figure 1).

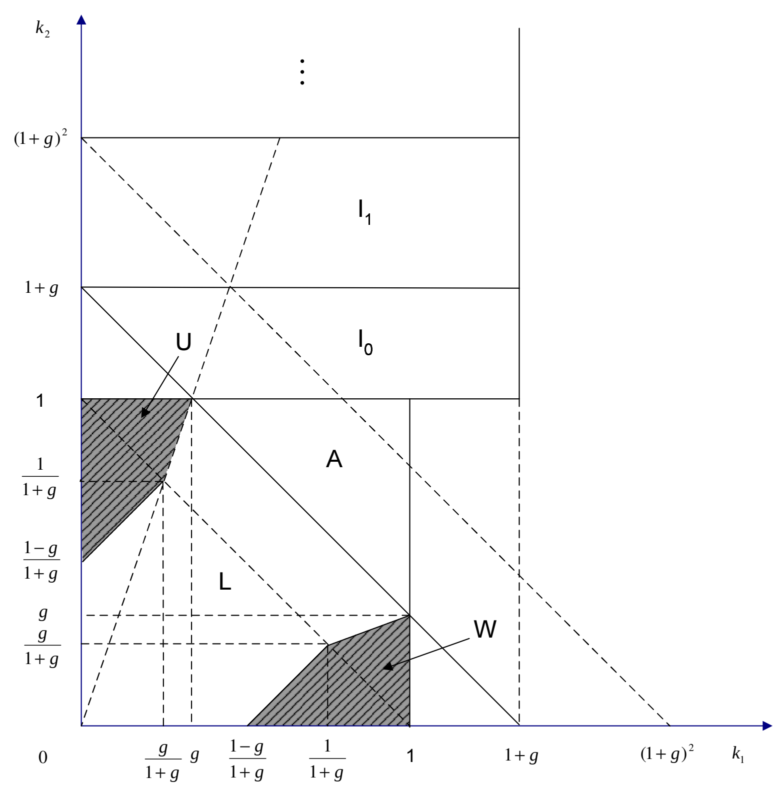

FIGURE 1: Regions for y^*.

We now define a symmetric strategy profile y^* with the property that along its (stochastic) outcome path, detrended capacity is eventually trapped in the region L. Moreover, in the long run, the firms converge to a situation where they share the market equally and have the same capacity. The structure of the strategy is relatively simple, though its formal description requires a decomposition into many regions. In region L, the firms have insufficient capacity to cover a possible demand growth. Here they are required to invest equally to bring the total capacity up to $1+g$. If demand grows next period, the firms are able to extract monopoly rents, and if demand does not grow, the excess capacity next period is g (a relatively small amount) and the firms are able to extract close to monopoly rents anyway. In region A and in $(1+g,\infty)\times(1+g,\infty)$, the firms make no investments, letting demand gradually catch up with the total capacity. Regions U and W play a special role and their odd geometry is dictated by the contraction of the detrended stock vector when demand grows. In region U (region W) only firm 1 (firm 2) makes an investment. The investment brings total capacity up to $1+g$, so that the detrended capacity stock falls into L if demand grows, and stays in A otherwise. When firm 2 has overcapacity (i.e., can cover the whole demand by itself) while firm 1 has insufficient capacity (region I), y^* lets firm 1 'exploit its monopoly power'. In region I, firm 2 'waits' until its detrended capacity falls below 1 (i.e., until demand increases above its current capacity). Meanwhile, ideally, firm 1 wants to increase capacity to exactly meet demand every period. Since demand growth is random, it might be optimal to invest only enough to cover current demand (especially if θ is relatively low). But if the overcapacity of firm 2 is small, firm 1 may find it optimal to invest even less. The reason is that when firm 2's overcapacity is small, it may take just a few periods for demand to grow enough so that the detrended stock falls in the square $[0,1]\times[0,1]$ and firm 2 starts competing for market share again. Moreover, while the stock k is in region A, firm 1's profit is $m(1-k_2)$, independent of k_1, and the larger is k_1, the longer it takes for the stock to fall into the region L. Therefore, when firm 2's overcapacity is small, an expensive investment for firm 1 is only profitable for a few periods, and firm 1 makes only a small investment to partially exploit its market power now without increasing too much the number of periods the stock stays in region A. In the proof of Theorem 1 below we will find an integer $\bar{r}\geq 0$ and construct two functions $\tau^*:\mathbf{N}\to\{0,1\}$ and $\hat{\tau}:\{0,1,\ldots,\bar{r}-1\}\to\mathbf{N}$ such that

(i) τ^* is weakly increasing.

(ii) $\hat{\tau}(r+1)\geq\hat{\tau}(r)+1$ for all $0\leq r<\bar{r}-1$.

When $k\in I_r$ and $r\geq\bar{r}$, firm 1 makes a 'full investment' equal to $(1+g)^{\tau^*(r)}-k_1$, so that its capacity increases to $(1+g)^{\tau^*(r)}$. Since $\tau^*(r)$ is either 0 or 1, a full investment makes firm 1's detrended capacity equal to 1 or $1+g$. If $k\in I_r$ and $0\leq r<\bar{r}$, firm 1 makes a 'partial investment' of no more than $(1+g)^{\hat{\tau}(r)}-k_1-k_2$. In no case will firm 1 invest more than $(1+g)^{\tau^*(r)}-k_1$, and there might be cases where $(1+g)^{\hat{\tau}(r)}-k_1-k_2>(1+g)^{\tau^*(r)}-k_1$. Hence, the partial investment is in general defined by

$$\bar{y}_1(k)=\min\{(1+g)^{\hat{\tau}(r)}-k_1-k_2,(1+g)^{\tau^*(r)}-k_1\}.$$

For each $k = (k_1, k_2) \geq 0$ let

$$y_1^*(k) = \begin{cases} \frac{1}{2}[1+g-k_1-k_2] & \text{if } k \in L \\ 1+g-k_1-k_2 & \text{if } k \in U \\ \bar{y}_1(k) & \text{if } k \in I_r,\ k_1+k_2 < (1+g)^{\hat{\tau}(r)} \text{ and } 0 \leq r < \bar{r} \\ (1+g)^{\tau^*(r)} - k_1 & \text{if } k \in I_r,\ k_1 < (1+g)^{\tau^*(r)} \text{ and } r \geq \bar{r} \\ 0 & \text{in all other cases,} \end{cases} \tag{1}$$

and $y_2^*(k) = y_1^*(k_2, k_1)$.

Note that when $k \in [0,1]^2$ this strategy prescribes positive investments only if aggregate capacity is insufficient to cover a possible demand growth, that is, only if $k_1 + k_2 < 1 + g$, and then aggregate investment is exactly $1 + g - k_1 - k_2$.

Definition: Let $B = [1 - \beta(1-\theta)]^{-1}$ and $\eta = B\beta\theta$, and define the functions (B and η are also functions of θ):

$$\psi(\theta) = \frac{1}{\eta} \qquad \text{and} \qquad \phi(\theta) = \frac{2-\beta}{\theta(2-\beta) + (1-\theta)\beta}.$$

Note that $0 < \eta < 1$ for any $\theta \in [0,1]$. Also recall that $\rho = [1-\beta]/\beta$.

Theorem 1: *If $\rho\phi(\theta) < m/\kappa < \rho\psi(\theta)$, the strategy $y^*(k)$ is an MPE.*

Proof: See the Appendix.

In the proof of Theorem 1 (in the Appendix), we explicitly derive the equilibrium value associated with each starting capital stock k. When $k \in L$, the equilibrium value for firm 1 increases with the difference $k_1 - k_2$ (see equation (4)). The upper bound on m/κ ensures that the temptation to increase this difference and become more dominant is less than the investment cost. On the other hand, firm 1 may be tempted to underinvest and save some investment cost. However, such a move allows firm 2 to increase its size (relative to firm 1) in future periods, and that hurts the continuation value of firm 1 for the same reason. The lower bound on m/κ guarantees that the savings in investment cost is less than the future losses. The equilibrium value function is piecewise linear, and the marginal value for firm 1 of overinvesting is strictly less than that of underinvesting. That is the reason there is a range of values for m/κ for which the firms have the proper incentives to make the investments prescribed by y^*.

Note that $\psi(\theta) > \phi(\theta) \geq 1$ and both are strictly decreasing in θ. When $\theta = 1$, $B = 1$, $\eta = \beta$, and $\psi(1) = \phi(1) = 1$. Therefore, when $\theta = 1$, Theorem 1 holds for $m/\kappa \in [\rho, \rho/\beta] = [\rho, \rho(1+\rho)]$. The upper bound for m/κ is very restrictive in this case. When demand grows with small probability (e.g. for low values of θ), the range of admissible values for m/κ expands considerably. As $\theta \downarrow 0$, $y^*(k)$ is an equilibrium for any $m/\kappa > \rho[2-\beta]/\beta = \rho(1+2\rho)$. For low values of θ, when $k \in L$, firm 1 (or firm 2) has little incentive to invest beyond $y_1^*(k)$ and arrive at a capital stock k' with excess capacity (that is, where $k_1' + k_2' > 1$). At such stock, firm 1's net revenue is $m(1 - k_2')$, independent of k_1', and it may take a long time before demand catches up with the total installed capacity.

Recall that for any initial capital stock, the stochastic detrended capacity stock trajectory generated by y^* is eventually trapped in the region L. When the firms follow y^*, if $k^t \in L$, in period $t+1$ with probability θ there is no excess capacity (that is $k_1^{t+1}+k_2^{t+1} \leq 1$), and with probability $1-\theta$ there is detrended excess capacity equal to g. Therefore, in the long-run, the average of $(K_t^1 + K_t^2)/D_t$ is $E^* = (1-\theta)g > 0$. That is, E^* is the average fraction (with respect to demand) of excess capacity.

5.2 *An Equilibrium without Security of Supply*

For relatively low values for the probability of growth, the firms can do better than in the equilibrium analyzed in the previous section. When capacity matches current demand and demand growth is very unlikely, investments are very likely to produce excess capacity. In this section, we study a strategy where the firms take a "conservative" approach to investing. Instead of guaranteeing that the demand growth is always covered, joint investment will now only be enough to cover *current* demand levels. As a result, this strategy will produce periodic (stochastic) rationing. Again, we identify sufficient conditions on the primitives of the model for this strategy to form an equilibrium. As we shall see in Theorem 2 below, this strategy is *not* an equilibrium when both the probability of demand growth and the ratio m/κ are relatively high. However, for low values of the probability of demand growth, this strategy is indeed an equilibrium for a wide range of values for m/κ.

The strategy is defined as follows:

$$\hat{y}_1(k) = \begin{cases} \frac{1}{2}[1-k_1-k_2] & \text{if } k_1+k_2 \leq 1 \\ \bar{y}_1(k) & \text{if } k \in I_r,\ k_1+k_2 < (1+g)^{\hat{\tau}(r)} \text{ and } 0 \leq r < \bar{r} \\ (1+g)^{\tau^*(r)} - k_1 & \text{if } k \in I_r,\ k_1 < (1+g)^{\tau^*(r)} \text{ and } r \geq \bar{r} \\ 0 & \text{in all other cases.} \end{cases} \tag{2}$$

This strategy differs from y^* in an important way. When $k \in [0,1)\times[0,1)$ and $k_1+k_2 < 1+g$, $y_1^*(k) + y_2^*(k) = 1+g-k_1-k_2$, guaranteeing that demand is fully covered the next period. By contrast, when $k_1+k_2 < 1$, $\hat{y}_1(k)+\hat{y}_2(k) = 1-k_1-k_2$, and when $1 \leq k_1+k_2 < 1+g$, $\hat{y}(k) = 0$. Therefore, if demand grows, the capacity is insufficient the next period and there is rationing. For a given ratio m/κ, the strategy $\hat{y}$ is an equilibrium when the probability θ of demand growth is relatively low.

Theorem 2: *The strategy $\hat{y}(k)$ is an MPE when*

$$\frac{2-\eta}{\eta}\frac{1-\beta}{2-\beta} > \frac{m}{\kappa} \geq \frac{1-\beta}{\beta} = \rho.$$

Proof: *See the Appendix.*

When the firms follow the strategy $\hat{y}$, it is easy to see that in the long run, in every period, with probability θ there is rationing and with probability $1-\theta$ installed capacity matches demand. Therefore, in the long-run, the average of $(D_t - K_1^t - K_2^t)/D_t$ is $\hat{E} = \theta g/(1+g)$. That is, $1-\hat{E}$ is the average fraction of covered demand.

Note that as $\theta \to 1$, $\eta \to \beta$ and the feasible interval for m/κ shrinks to the singleton $\{\rho\}$. The intuition is clear: as θ increases, a demand growth is more likely, and each firm becomes tempted to increase its investments by g (so that total capacity increses to $1+g$) to capture additional rents in the next (and future) period(s).

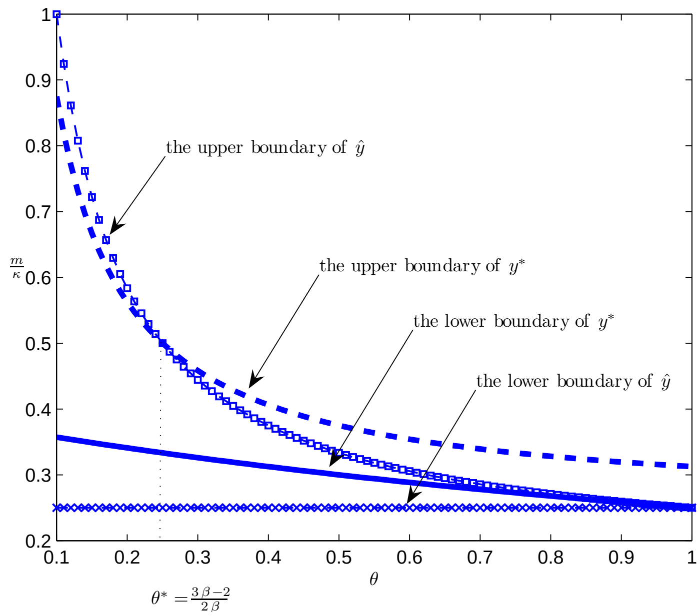

FIGURE 2: Upper and Lower bounds for y^* and $\hat{y}$.

For relatively low values of θ, both strategy profiles y^* and $\hat{y}$ are MPE for a wide-range of values for m/κ (see Figure 2).

5.3 *Welfare Comparison*

For a proper welfare analysis, we need information about the consumers' willingness to pay. We have made the assumption that demand is perfectly inelastic. This assumption implicitly captures the consumers' reaction to the indirect market mechanisms in place. Current regulation (or a contractual agreement) does not allow the retailers to instantaneously reflect in consumer prices the changes in the spot price. Thus, the demand function we have assumed does not properly capture the consumers' willingness to pay – demand is assumed to be inelastic precisely because while spot prices are changing, the consumer prices have been set ahead of time. To estimate a demand function that accurately represents the consumers' marginal willingness to pay is a delicate exercise (see, for example, Goett et al. (1988)). For our purposes, however, it will suffice to assume that the marginal willingness to pay is a decreasing function of the quantity consumed. For simplicity, we now assume that the marginal willingness to pay function is given by $p = P - \sigma q$, where q is the quantity consumed, and P and σ are two positive constants. The slope σ decreases randomly over time. If σ_t is the slope of the marginal willingness to pay function in period t, then in period $t+1$, $\sigma_{t+1} = \sigma_t/(1+g)$ with probability θ and $\sigma_{t+1} = \sigma_t$ with probability $1-\theta$. Without loss of generality we normalize variables so that $\sigma_0 = 1$.

Retailers buy electricity from the producers at the spot price and sell it to the consumers at a previously contracted price p_t^r. Thus, (short-run) demand is independent of the spot price; the quantity demanded is $D_t = [P - p_t^r]/\sigma_t$. In the long-run, p_t^r is set equal to the average spot price (so retailers make 0 profits).

In the previous sections we multiply quantities and capacities in period t by the factor D_0/D_t to obtain detrended variables. We also normalized $D_0 = 1$. But the magnitude of demand depends on the equilibrium we study. As we compare different equilibria, we can no longer normalize $D_0 = 1$ for all of them (and we choose instead to normalize $\sigma_0 = 1$). The equivalent detrending is obtained here by multiplying quantities and capacities in period t by the factor $\sigma_t/\sigma_0 = \sigma_t$.

When the average spot price is p_t^r and capacity exceeds D_t, the producers revenues are $R_t = (p_t^r - c)D_t$ and consumer surplus is $CS_t = D_t(P - p_t^r)/2$. The corresponding detrended revenues and consumer surplus are $r_t = (p_t^r - c)[P - p_t^r]$ and $cs_t = [P - p_t^r]^2/2$. When D_t exceeds capacity, we will assume that rationing favors the consumers with the highest willingness to pay. This assumption effectively underestimates the welfare losses due to rationing. Let $p_t^k = P - \sigma_t(K_1^t + K_2^t)$ be the marginal willingness to pay when there is rationing and the quantity supplied is $K_1^t + K_2^t$. Then $p_t^k > p_t^r$, $R_t = (p_t^r - c)[K_1^t + K_2^t]$, and $CS_t = [K_1^t + K_2^t](p_t^k - p_t^r) + [K_1^t + K_2^t](P - p_t^k)/2 = [K_1^t + K_2^t](P + p_t^k - 2p_t^r)/2$.

In the equilibrium y^*, in the long run, each period capacity matches demand exactly with probability θ and capacity exceeds demand with probability $1-\theta$. In the former case, the spot price is equal to $\bar{p}$, and in the latter case the expected spot price is $(1-g)\bar{p} + gc$. Hence, in the long-run, the average spot price is $p^r = \theta\bar{p} + (1-\theta)[(1-g)\bar{p} + gc] = \bar{p} - (1-\theta)gm$ and the detrended average demand is $d = [P - \bar{p} + (1-\theta)gm] = [P - (1-(1-\theta)g)m - c]$. Also, the detrended average investment is $\theta g d$ per period. Let $\xi = 1 - (1-\theta)g$. Thus, the long-run average detrended consumer surplus, industry revenues and total surplus are:

$$cs^* = \frac{1}{2}[P - c - \xi m]^2, \qquad r^* = \xi m[P - c - \xi m]$$
$$s^* = \frac{1}{2}[(P-c)^2 - (\xi m)^2] - \kappa\theta g[P - c - \xi m].$$

In the equilibrium $\hat{y}$, the spot price is $\bar{p}$ in every period. Therefore $\hat{p}^r = \bar{p}$ and $\hat{d} = P - \bar{p}$. In average, the long-run detrended capacity is $\hat{d}(1-\hat{E}) = \hat{d}[1 - \theta g/(1+g)]$ per period. Thus, $\hat{p}^k = P - \hat{d}[1 - \theta g/(1+g)] = \bar{p} + \hat{d}\theta g/(1+g)$. The detrended average investment is $\theta g\hat{d}/(1+g)$. Let $\hat{\xi} = 1 + (1-\theta)g$. Therefore, the long-run average detrended consumer surplus and industry revenues are:

$$\widehat{cs} = \frac{\hat{\xi}}{2}\left[\frac{P-\bar{p}}{1+g}\right]^2(\hat{\xi} + 2\theta g), \qquad \hat{r} = \hat{\xi}m\left[\frac{P-\bar{p}}{1+g}\right]$$
$$\hat{s} = \frac{\hat{\xi}}{2}\left[\frac{P-\bar{p}}{(1+g)^2}\right][(P-\bar{p})(1+g+\theta g) + 2m(1+g)] - \kappa\theta g\left[\frac{P-\bar{p}}{1+g}\right].$$

The producers prefer the equilibrium $\hat{y}$ to the equilibrium y^* when

$$(\bar{p}-c)(1-\theta)[1-(1-\theta)g] < \left[\frac{P-\bar{p}}{1+g}\right][1+g-\theta(2+g)] + \kappa\theta g\left[\frac{1}{1+g}\left[\frac{P-\bar{p}}{\bar{p}-c}\right] + 1 - \theta\right].$$

This inequality is possible only when θ is relatively small (for the inequality to hold, it is also necessary that $\bar{p}$ is in the lower half of the interval $[c, P]$). However, for all parameters, $cs^* > \widehat{cs}$ and $s^* > \hat{s}$. There are two reasons for the latter. In the absence of rationing, total surplus with price $\hat{p}^r = \bar{p}$ (call this $\tilde{s}$) is smaller than with price p^r because $c < p^r < \hat{p}^r$. That is $\tilde{s} < s^*$. In addition, $\hat{s} < \tilde{s}$ because with rationing there are further surplus losses.

As remarked above, among equilibria that always cover demand, total surplus is larger the smaller is the average spot price. Roughly, the average spot price is decreasing in the average fraction of excess capacity (AFEC). Clearly y^* minimizes the AFEC among all the strategies that ensure security of supply. On the other hand, maintainig a higher AFEC is wasteful: the detrended average cost of a detrended average capacity k is $\kappa\theta gk$. It is possible that there are other MPE's that maintain a higher AFEC and a lower average spot price, and therefore produce more social surplus, even though they incur additional investment costs. We do not know if such an MPE exists. It would also be desirable to design regulatory mechanisms that induce such MPE's. We do not pursue this question here, though we think it is extremely important.

Alternatively, another way to increase total welfare is to reduce m (see the expression for s^* above). This can be accomplished directly by reducing the cap price $\bar{p}$. Usually, the cap price $\bar{p}$ is adjusted to approximately match the marginal (or average) willingness to pay when there is rationing. Of course, in this case, $\bar{p}$ depends on the severity of the average rationing that is considered. Our welfare analysis of the MPE y^* suggests that an alternative definition of $\bar{p}$, one that has little to do with the marginal willingness to pay, would be more appropriate. To maximize the total welfare s^*, we need to minimize $\bar{p}$ subject to the constraints that the firms make nonnegative profits and that y^* remains an equilibrium. The firms make nonnegative profits when $r^* - \kappa\theta gd \geq 0$. This condition is equivalent to $\bar{p} \geq c + \kappa\theta g/[1-(1-\theta)g]$. For y^* to be an equilibrium it must be that $m/\kappa > \rho\phi(\theta)$. That is

$$\bar{p} > c + \kappa\frac{(1-\beta)(2-\beta)}{\theta(2-\beta)+(1-\theta)\beta}.$$

Therefore, to maximize welfare with the MPE y^*, we should set

$$\bar{p} = c + \kappa \cdot \max\left\{\frac{\theta g}{1-(1-\theta)g}\ ,\ \frac{(1-\beta)(2-\beta)}{\theta(2-\beta)+(1-\theta)\beta}\right\}.$$

5.4 *Deterministic Demand Growth*

When $\theta = 1$, the range of values for m/κ for which $\hat{y}$ is an MPE is empty. Also, when $m/\kappa > \rho + \rho^2$, y^* is no longer an MPE because the firms have an incentive to overinvest when $k_1 + k_2 \leq 1$. The reason for this breakdown is that the continuation values for y^* (for $\hat{y}$) when the current capital stock $k \in [0,1]^2$ is such that $k_1 + k_2 > 1 + g$ $(k_1 + k_2 > 1)$ are too attractive.

We next modify the strategy of Theorem 1 to decrease the firms' payoffs when the initial state $k^0 \in (0, 1+g]^2$ is such that $k_1^0 + k_2^0 > 1$. When there is overcapacity, that strategy requires that the firms make no investments to allow the demand to catch up with the installed capacity. Now, instead, we require the firms to maintain overcapacity in every period and the stock trajectory remains trapped in the region where $k_1^t + k_2^t > 1$ for all $t \geq 1$. For any such initial stock k^0 that is asymmetric, that is $k_1^0 \neq k_2^0$, each firm increases its capacity to *almost* $1 + g$. Thereafter, the firms allow the stock to slowly decrease to the state $(\frac{1}{2}(1+g), \frac{1}{2}(1+g))$. More precisely, $k_1^t = k_2^t > k_1^{t+1} = k_2^{t+1}$ for all $t \geq 1$ and $k_i^t \to \frac{1}{2}(1+g)$. This modified strategy relaxes the temptation to overinvest and is an MPE when $\theta = 1$ for relatively high values of m/κ.

For $\epsilon > 0$, let

$$\begin{aligned}
L &= \{(k_1, k_2) \mid k_1 + k_2 \leq 1\} \cup \{(k, k) \mid \tfrac{1}{2} < k \leq \tfrac{1}{2}(1+g)\} \\
M &= \{(k, k) \mid \tfrac{1}{2}(1+g) < k \leq 1\} \\
A(\epsilon) &= \{(k_1, k_2) \mid k_1 + k_2 > 1 \text{ and } \epsilon(1+g) < k_i \leq (1-\epsilon)(1+g),\ i = 1, 2\} \\
U(\epsilon) &= \{(k_1, k_2) \mid k_1 + k_2 > 1,\ 0 < k_1 \leq \epsilon(1+g), \text{ and } 0 < k_2 < 1+g\} \\
W(\epsilon) &= \{(k_1, k_2) \mid k_1 + k_2 > 1,\ 0 < k_2 \leq \epsilon(1+g), \text{ and } 0 < k_1 < 1+g\},
\end{aligned}$$

and define $A^\circ(0) = \operatorname{int}(A(0))$. Clearly, for $0 < \epsilon_1 < \epsilon_2 < (1+2g)/(2+2g)$, $A^\circ(0) \supset A(\epsilon_1) \supset A(\epsilon_2) \neq \emptyset$. For $\bar{\epsilon} > 0$ is to be determined later, define the function $\epsilon : A^\circ(0) \to (0, 1)$, as follows:

$$\epsilon(k) = \begin{cases} \bar{\epsilon} & \text{if } k \in A(\bar{\epsilon}) \\ \frac{k_1}{2(1+g)} & \text{if } k \in U(\bar{\epsilon}) \\ \frac{k_2}{2(1+g)} & \text{if } k \in W(\bar{\epsilon}) \\ 1 - \max\{k_1, k_2\}/(1+g) & \text{if } k \in A^\circ(0) \backslash [A(\bar{\epsilon}) \cup U(\bar{\epsilon}) \cup W(\bar{\epsilon})]. \end{cases}$$

We now consider the strategy:

$$y_1^*(k) = \begin{cases} \frac{1}{2}[1 + g - k_1 - k_2] & \text{if } k \in L \\ (1 - \epsilon(k))(1+g) - k_1 & \text{if } k \in A^\circ(0) \backslash \{(k, k) \mid \frac{1}{2} < k \leq 1\} \\ \frac{1}{2} g(1+g) & \text{if } k \in M \\ 1 + g - k_1 & \text{if } k_1 < 1 + g \text{ and } k_2 \geq 1 + g \\ 0 & \text{in all other cases} \end{cases} \tag{3}$$

and $y_2^*(k) = y_1^*(k_2, k_1)$.

The strategy in Theorem 1 adjusts total capacity to exactly meet demand next period whenever $k_1 + k_2 \leq 1 + g$. The current strategy does the same in the smaller region L (line 1). When $k \in A^\circ(0) \backslash M$ and $k_1 \neq k_2$, the firms invest so their capacities next period equal are $(1 - \epsilon(k), 1 - \epsilon(k))$ (line 2). The idea here is that next period the firms enjoy revenues close to 0. The larger is the capacity of each firm, the smaller are the revenues, and so we would like to make their capacities equal to 1. However, because the revenue function is discontinuous at the boundary $\{1\} \times [0, 1] \cup [0, 1] \times \{1\}$, asking the firms to bring their capacity stock to exactly $(1, 1)$ is not feasible: each firm would then have a strong incentive not to invest at all. Indeed, if a firm expects the opponent's capacity to be 1 next period, then it wants to keep its own capacity strictly below 1 so it can extract monopoly rents next period. Thus, y^* requires instead that each firm brings its capacity to $1-\epsilon(k)$ next period. The function $\epsilon(k)$ is constructed so that for $k \in A^\circ(0)$: (i) $\epsilon(k) \leq \bar{\epsilon}$; (ii) there exist nonnegative investments that make tomorrow's capital stock equal to $(1-\epsilon(k), 1-\epsilon(k))$; and (iii) even if only one firm follows the investment strategy y^* while the other makes no investment, the total capacity next period exceeds 1. When $k \in A(\bar{\epsilon})$, since $k_i \leq (1-\bar{\epsilon})(1+g)$, $i = 1, 2$, to make tomorrow's capital stock equal to $(1-\bar{\epsilon}, 1-\bar{\epsilon})$, each firm needs to make a nonnegative investment today. And since $k_i > \bar{\epsilon}(1+g)$, $i = 1, 2$, if firm 1 makes no investement and firm 2 follows y^*, tomorrow's total capacity is $1-\bar{\epsilon}+k_1/(1+g) > 1$. When $k \in U(\bar{\epsilon})$, $\epsilon(k) = k_1/[2(1+g)] \leq \bar{\epsilon}/2$. Also, if firm 1 makes no investment and firm 2 follows y^*, tomorrow's total capacity is $1+k_1/[2(1+g)] > 1$. Note that when $k_1 = 0$, the strict inequality does not attain.[5] When firm 2 makes no investment and firm 1 follows y^*, the total capacity is even more. The situation is similar in the region $W(\bar{\epsilon})$. Finally, let $k \in A^\circ(0) \backslash [A(\bar{\epsilon}) \cup U(\bar{\epsilon}) \cup W(\bar{\epsilon})]$. For example, assume that $(1-\bar{\epsilon})(1+g) < k_2 < 1+g$ and $\bar{\epsilon}(1+g) < k_1 \leq (1-\bar{\epsilon})(1+g)$. Then, $\epsilon(k) = 1-k_2/(1+g) < \bar{\epsilon}$, and the capital stock $(1-\epsilon(k), 1-\epsilon(k))$ is reached when firm 1 makes a positive investment and firm 2 makes no investment. Also, when firm 1 makes no investment while firm 2 follows y^*, tomorrow's total capacity is $1-\epsilon(k)+k_1/(1+g) > 1-\bar{\epsilon}+\bar{\epsilon} = 1$.

When $(k_1, k_2) \in M$, the firms let their detrended capacity fall 'slowly' towards the symmetric capacity stock $(\frac{1}{2}(1+g), \frac{1}{2}(1+g))$ (line 3). This feature of the strategy is remarkable in that along this symmetric capacity stock trajectory, the firms maintain excess capacity in *every* period. Consequently, this trajectory has a relatively low total payoff for the firms. Finally, in the region where $k_1 < 1+g$ and $k_2 \geq 1+g$, firm 2 makes no investment (line 5) and firm 1 invests to get its capacity equal to 1 and enjoys monopoly profits next period (line 4).

THEOREM 3: *Assume that* $\theta = 1$, $k^0 > (0,0)$, $\beta > \frac{1}{2}$ *and*

$$\frac{m}{\kappa} > \rho \left[\frac{2\rho}{g(1-g)} - 1 \right].$$

Then there exists $\bar{\epsilon} > 0$ *such that the strategy* y^* *is a Markov perfect equilibrium.*

PROOF: See the Appendix.

Our main motivation to construct the equilibrium y^* of Theorem 3 was to demonstrate that even when the probability of demand growth is high, there are equilibria like that of Theorem 1 that sustain no excess capacity (in the long run). In fact, with the equilibrium y^* of Theorem 3, what happens in the long run depends on the initial capacity stock k^0. When $k_1^0+k_2^0 \leq 1$, total capacity matches demand in every period along the equilibrium path. But when $k_1^0+k_2^0 > 1$, k^t approaches in the long run the capacity stock $(\frac{1}{2}(1+g), \frac{1}{2}(1+g))$, and excess capacity exceeds g all the time. As we discussed in the previous section, maintainig excess capacity is wasteful, but also reduces spot prices and increases consumer surplus.

[5] To avoid this possibility, in Theorem 3 below we assume that $k^0 > (0,0)$, so that $k^t > (0,0)$ for all $t \geq 1$ and for any investment strategy.

6. Conclusions

Markets for electric power are often regulated with the aim of achieving efficiency and security of supply. Analyzing a model that incorporates some of the essential features of common regulatory schemes, we show that both goals maybe poorly served. The model makes specific institutional assumptions about the spot market, along with other simplifying assumptions. Nevertheless, the analysis captures the main driving forces and a conclusion that is likely to be robust. The firms can extract (almost) monopoly rents by keeping excess capacity to a minimum. A firm's share of the rents is proportional to its own capacity, and hence each firm has an incentive to increase its market share. However, investing too much in an attempt to grab market share drastically reduces the spot price and the rents. Moreover, as demand grows continuously, market share gains are only temporary and can be easily eroded by the other firms in future periods. Therefore, the incentive to maintain monopoly prices is dominant and total capacity rarely exceeds demand. In certain cases (Theorem 2), capacity is insufficient and there is rationing. These results attain even though we restrict attention to 'non-collusive' Markovian equilibria. The problem is only likely to be exacerbated in collusive equilibria where the firms use intertemporal incentives to punish market grabbers.

While investment is efficient in the sense that there are no wasteful investments, our equilibria reveal key deficiencies in the electricity markets we model. The spot market auction does not elicit the healthy competition it was designed for. The 'price inelasticity' of demand leaves the consumers vulnerable to serious exploitation by the firms. Of course, our model does not include capacity payments (i.e., compensation for installed capacity), that are often provided by the regulatory agencies, in part as a mechanism to mitigate this problem. It would be interesting to study how effective these additional incentives are, and our model provides a realistic framework to do that. We do not consider explicitly the possibility of entry. However, since we allow any initial state, our model does indirectly cover the case where a second firm enters the market. We can capture this by initializing the second firm's capacity stock at 0. In our equilibria, the 'entry' of the second firm does not affect the nature of the outcome (prices, for example, remain monopolistic). We conjecture that a similar outcome will attain if we allow the entry of a third firm (for a range of parameters). However, we expect that as the number of firms increases, eventually our equilibria will no longer be sustainable and perhaps the firms will have to maintain excess capacity.

7. Appendix

7.1 *Proof of Theorem 1*

We first derive that the expected discounted payoff for firm 1 when the initial capital stock is $k \in L$ and the firms follow the investment strategy y^*. To begin, assume that $k \in L$ and that $k_1 + k_2 = 1$. Let $\Delta = k_1 - k_2$. Then, $k_1 = (1+\Delta)/2$ and $k_2 = (1-\Delta)/2$. Therefore, in period 1, capacity stock is $\frac{1}{2}(1+g+\Delta, 1+g-\Delta)$, and demand is $1+g$ with probability θ and 1 with probability $1-\theta$. Recall that when total capacity matches demand, firm 1's revenue is proportional to its own capacity, and when there is excess capacity, firm 1's revenue is proportional to the complement of firm 2's capacity. Let $G = \theta(1+g) + (1-\theta)(1-g) = 1 + (2\theta - 1)g$. Then, firm 1's net profit in period 0 is $\frac{1}{2}[m(1+\Delta) - \kappa g]$ and expected revenue in period 1 is

$$\theta \frac{m}{2}[1+g+\Delta] + (1-\theta)\frac{m}{2}[1-g+\Delta] = \frac{m}{2}[\Delta + G].$$

In general, in period t, the expected demand is $(1+\theta g)^t$, with probability θ capacity matches demand exactly, and with probability $1-\theta$ there is excess capacity. When capacity matches demand, each firm invests $D_t g/2$, where D_t is current demand. Hence, firm 1's expected discounted payoff is

$$\begin{aligned} v_1(k|y^*) &= \frac{1}{2}[m(1+\Delta) - \kappa g] + \sum_{t=1}^{\infty} \frac{\beta^t}{2}\Big[m[\Delta + (1+\theta g)^t G] - \theta\kappa(1+\theta g)^t g\Big] \\ &= H + \frac{m\Delta}{2(1-\beta)}, \quad \text{where} \quad H = \frac{g}{2}[m(1-2\theta) - \kappa(1-\theta)] + \frac{mG - \kappa\theta g}{2(1-\gamma)} \end{aligned}$$

is a constant independent of k.

For general $k \in L$ that does not necessarily satisfy $k_1 + k_2 = 1$, let $\Delta = k_1 - k_2$ and $O = 1 - (k_1 + k_2)$. Define $\tilde{k}_i = k_i + O/2$ for $i = 1, 2$. Then $\tilde{k}_1 + \tilde{k}_2 = 1$ and $\tilde{k}_1 - \tilde{k}_2 = k_1 - k_2 = \Delta$. Hence $v_1(k|y^*)$ coincides with $v_1(\tilde{k}|y^*)$ except for the payoffs in period 0:

$$v_1(k|y^*) = v_1(\tilde{k}|y^*) - \left[m\tilde{k}_1 - \frac{\kappa g}{2}\right] + [r_1^*(k) - \kappa y_1^*(k)].$$

Since $g/2 = y_1^*(\tilde{k})$, we have that $g/2 - y_1^*(k) = [(k_1 + k_2) - (\tilde{k}_1 + \tilde{k}_2)]/2 = -O/2$. Also, $r_1^*(k) = mk_1$ if $O > 0$, and $r_1^*(k) = m(1 - k_2)$ if $O < 0$. Since $\tilde{k}_1 = 1 - \tilde{k}_2$, $r_1^*(k) - m\tilde{k}_1 = -m|O|/2$. Replacing these two expressions into the equation above we obtain

$$v_1(k|y^*) = H + \frac{1}{2}\left[\frac{m\Delta}{1-\beta} - m|O| - \kappa O\right]. \tag{4}$$

We next compute the marginal value of initial capacity in various regions. Clearly, the value function is only piece-wise differentiable and it has 'kinks'.

Region L: From (4), for each $k \in L$,

$$\frac{\partial v_1}{\partial k_1}(k|y^*) = \begin{cases} \frac{1}{2}[m(2-\beta)/(1-\beta)+\kappa] & \text{if } k_1+k_2<1 \\ \frac{1}{2}[m\beta/(1-\beta)+\kappa] & \text{if } 1<k_1+k_2<1+g. \end{cases} \tag{5}$$

Later we will also require the derivative of the function $\mu(x) = v_1(k_1+x, k_2-x|y^*)$ at $x = 0$:

$$\mu'(0) = \lim_{x\to 0} \frac{m[(k_1+x)-(k_2-x)] - m[k_1-k_2]}{2(1-\beta)x} = \frac{m}{1-\beta}. \tag{6}$$

Region A: We subdivide A into 'stripes'. Let $k \in [0,1)\times[0,1)$ and $\tau \in \mathbf{N}$ be such that $(1+g)^\tau < k_1+k_2 < (1+g)^{\tau+1}$. Define $k' = k/(1+g)$ and $k'' = k$. Then

$$v_1(k|y^*) = m(1-k_2) + \beta[\theta(1+g)v_1(k'|y^*) + (1-\theta)v_1(k''|y^*)] \tag{7}$$

or

$$v_1(k|y^*) = B[m(1-k_2) + \beta\theta(1+g)v_1(k'|y^*)].$$

Using (7) repeatedly, we obtain (recall that $\eta = B\beta\theta$)

$$v_1(k|y^*) = C(k_2) + [\eta(1+g)]^\tau v_1(\hat{k}|y^*),$$

where $\hat{k} = k/(1+g)^\tau$ and $C(k_2)$ is a function of k_2 only. Since $1 < \hat{k}_1 + \hat{k}_2 < 1+g$, (5) implies that

$$\frac{\partial v_1}{\partial k_1}(k|y^*) = [\eta(1+g)]^\tau \frac{\partial v_1}{\partial k_1}(\hat{k}|y^*)\frac{1}{(1+g)^\tau} = \frac{\eta^\tau}{2}\left[\frac{\beta m}{1-\beta}+\kappa\right]. \tag{8}$$

Region U: If $k \in U$, the capital stock at the end of period 0 is $(1+g-k_2, k_2)$, independent of k_1. Therefore, the marginal value of initial capital is just equal to the marginal value of capital in period 0. That is

$$\frac{\partial v_1}{\partial k_1}(k|y^*) = \begin{cases} m+\kappa & \text{if } k_1+k_2<1 \\ \kappa & \text{if } k_1+k_2>1. \end{cases} \tag{9}$$

Region W: Pick $k \in W$ such that $k_1+k_2 > 1$. With k' and k'' defined as $k_1' = k_1/(1+g)$, $k_2' = 1-k_1'$, and $k'' = (1+g)k'$, $v_1(k|y^*)$ satisfies equation (7). Therefore, from (6) we obtain

$$\frac{\partial v_1}{\partial k_1}(k|y^*) = \beta\theta(1+g)\left[\frac{m}{1-\beta}\right]\frac{1}{1+g} + \beta(1-\theta)\left[\frac{m}{1-\beta}\right] = \frac{\beta m}{1-\beta}. \tag{10}$$

Region I_r: The analysis for this region is lengthy and delicate because we simultaneously compute the marginal value of capital and derive the functions τ^* and $\hat{\tau}$ mentioned in the definition of y^* (equation (1)). For each pair of nonnegative integers r and τ, it will be convenient to define the regions

$$X(r,\tau) = \{k \in I_r \mid (1+g)^\tau < k_1 + k_2 < (1+g)^{\tau+1}\}.$$

Assume that the integer τ is such that $y^*(k) = (0,0)$ for all $k \in I_0$ such that $k_1 + k_2 > (1+g)^\tau$. Then, for any such k, we can compute firm 1's expected value using the following recursive equation:

$$v_1(k|y^*) = \begin{cases} mk_1 + \beta[\theta(1+g)v_1(k'|y^*) + (1-\theta)v_1(k|y^*)] & k_1 \leq 1 \\ \beta[\theta(1+g)v_1(k'|y^*) + (1-\theta)v_1(k|y^*)] & k_1 > 1, \end{cases}$$

where $k' = k/(1+g) \in [0,1) \times [0,1)$. Thus,

$$\frac{\partial v_1}{\partial k_1}(k|y^*) = \begin{cases} Bm + \eta \dfrac{\partial v_1}{\partial k_1}(k'|y^*) & \text{if } k_1 < 1 \\ \eta \dfrac{\partial v_1}{\partial k_1}(k'|y^*) & \text{if } k_1 > 1. \end{cases} \tag{11}$$

Having computed $v_1(k|y^*)$ under the assumption that $y^*(k) = (0,0)$ for all $k \in I_0$ such that $k_1 + k_2 > (1+g)^\tau$ (value iteration), we can now verify whether $y^*(k) = (0,0)$ is indeed optimal (policy iteration). Let $k \in X(0,\tau)$. If firm 1 assesses its current choice of investment assuming that in the future both firms will follow y^*, firm 1 faces the following optimization problem:

$$\max_{y_1 \geq 0} \left[-\kappa y_1 + \beta\Big[\theta(1+g)v_1\Big(\Big(\frac{k_1+y_1}{1+g}, \frac{k_2}{1+g}\Big) \,\Big|\, y^*\Big) + (1-\theta)v_1((k_1+y_1, k_2)|y^*)\Big]\right].$$

Let $\alpha(k)$ denote the derivative of the objective function at $y_1 = 0$ (or at any $0 \leq y_1 < (1+g)^{\tau+1} - k_1 - k_2$). Note that $\beta[\theta + \eta(1-\theta)] = \eta$ and that $\beta(1-\theta)B = \eta(1-\theta)/\theta$. Assume $k_1 < 1$. Then, using (11), we obtain

$$\alpha(k) = -\kappa + \beta\left[\theta \frac{\partial v_1}{\partial k_1}(k'|y^*) + (1-\theta)\frac{\partial v_1}{\partial k_1}(k|y^*)\right] = -\kappa + \eta\left[\frac{\partial v_1}{\partial k_1}(k'|y^*) + m\frac{(1-\theta)}{\theta}\right],$$

where $k' = k/(1+g)$. When $k_1 > 1$ the expression for $\alpha(k)$ is the same but with the term $m(1-\theta)/\theta$ removed. The actual value of $\alpha(k)$ depends on where in the square $[0,1) \times [0,1)$ does k' land. When $\tau = 0$ or when $\tau = 1$ and $k_2 > k_1/g$, for example, $k' \in U$ and we can use (9) to get an expression for $[\partial v_1/\partial k_1](k'|y^*)$. If $k_2 \leq k_1/g$, we can use (4) instead. Hence

$$\alpha(k) = \begin{cases} -\kappa + \eta[(m+\kappa) + m(1-\theta)/\theta] & \text{if } \tau = 0 \\ -\kappa + \eta[\kappa + m(1-\theta)/\theta] & \text{if } \tau = 1 \text{ and } gk_2 > k_1 \\ -\kappa + \eta^\tau \frac{1}{2}[\beta m/(1-\beta) + \kappa] + \eta m(1-\theta)/\theta & \text{if } \tau \geq 1,\ gk_2 \leq k_1 < 1 \\ -\kappa + \eta^\tau \frac{1}{2}[\beta m/(1-\beta) + \kappa] & \text{if } k_1 > 1. \end{cases} \tag{12}$$

Note that by lines 3 and 4, $\alpha(k)$ is a decreasing function of k_1 for $gk_2 < k_1$ (because τ increases with k_1 and $\eta < 1$). Therefore, if $\alpha(k) \leq 0$ for $k \in X(0,\tau)$, then $\alpha(k) \leq 0$ for all $k \in X(0, \tau+1)$.

Let $k \in X(0,0)$. Since $\frac{m}{\kappa} > \frac{1-\beta}{\beta}$ (by assumption), it follows (from line 1) that $\alpha(k)$ is a decreasing function of θ. Therefore, for any $\theta \in [0,1]$, $\alpha(k)$ is bounded below by the expression in the right hand side evaluated at $\theta = 1$. That is,

$$\alpha(k) \geq -\kappa + \beta(m+\kappa) = \kappa\beta\left[\frac{m}{\kappa} - \frac{1-\beta}{\beta}\right] > 0.$$

Therefore, by policy iteration, we conclude that $y_1^*(k) > 0$ (and therefore $y_1^*(k) \geq (1+g) - k_1 - k_2$) for all $k \in X(0,0)$.

Recall that $(\bar{r}, \tau^*, \hat{\tau})$ determines the level of investment of firm 1 for any $k \in I$. If $k \in I_r$, firm 1 makes full investments if $r \geq \bar{r}$ and partial investments if $r < \bar{r}$. In I_r, $r \geq \bar{r}$, firm 1 makes a 'full investment' for any $k \in I_r$ with $k_1 < (1+g)^{\tau^*(r)}$ and brings its capital stock up to $(1+g)^{\tau^*(r)}$ (that is, up to 1 if $\tau^*(r) = 0$, or up to $1+g$ if $\tau^*(r) = 1$). In I_r, $r < \bar{r}$, firm 1 makes a 'partial investment' of $(1+g)^{\hat{\tau}(r)} - k_1 - k_2$ for any $k \in I_r$ with $k_1 + k_2 < (1+g)^{\hat{\tau}(r)}$, so that after the investment, the total capital stock is $(1+g)^{\hat{\tau}(r)}$. A priori, we do not know $\bar{r}$, therefore we define $\tau^*(r)$ and $\hat{\tau}(r)$ for each r.

Since $y_1^*(k) > 0$ for all $k \in X(0,0)$, either $\bar{r} = 0$ or $\bar{r} \geq 1$ and $\hat{\tau}(0) \geq 1$.

Let $k \in I_0$ with $k_1 > 1$. Note that η is increasing in θ. Since $\eta \in [0,1)$, by line 4 of (12),

$$\alpha(k) \leq -\kappa + \frac{\eta}{2}\left(\frac{\beta m}{1-\beta} + \kappa\right) = \frac{\kappa(2-\eta)}{2}\left[\frac{m}{\kappa}\left[\frac{\eta\beta}{(2-\eta)(1-\beta)}\right] - 1\right] < 0$$

because by assumption

$$\frac{m}{\kappa} < \frac{1-\beta}{\eta\beta} < \frac{(2-\eta)}{\eta}\frac{(1-\beta)}{\beta}.$$

Therefore, define $\tau^*(0) = 0$. Let $\bar{\tau}$ be the index of the last diagonal $\{k \mid k_1 + k_2 = (1+g)^\tau\}$ that passes below the point $k = (1, (1+g))$. That is, $(1+g)^{\bar{\tau}} \leq 2 + g < (1+g)^{\bar{\tau}+1}$. Pick $k \in I_0$ such that $k_1 + k_2 > (1+g)^{\bar{\tau}}$ and $k_1 < 1$. If $\alpha(k) \geq 0$, then let $\bar{r} = 0$. Otherwise, $\bar{r} \geq 1$ and

$$\hat{\tau}(0) = \max\{\tau + 1 \mid \alpha(k) \geq 0 \ \forall k \in X(0,\tau) \text{ with } k_1 < 1\}.$$

This finishes the analysis of I_0.

We now proceed recursively to study I_{r+1}, $r \geq 0$. Suppose that we have already determined $\tau^*(j)$ and $\hat{\tau}(j)$ for $j = 0, \ldots, r$. Let τ be such that $y_1^*(k') > 0$ for all $k' \in X(r, \tau-1)$ (and hence for all $k' \in X(r,j)$, $j = 0, \ldots, \tau - 1$). Assume temporarily that $y^*(k) = (0,0)$ for all $k \in I_{r+1}$ with $k_1 + k_2 > (1+g)^\tau$. As before, under this assumption, for any such k, equation (11) is still valid. Again, let $\alpha(k)$ be the derivative of the corresponding investment optimization problem at the end of the period when firm 1 assumes that y^* will be followed in the future. Let $k \in X(r+1, \tau)$. Then

$$\alpha(k) = \begin{cases} -\kappa + \eta\left[\dfrac{\partial v_1}{\partial k_1}(k'|y^*) + m\dfrac{(1-\theta)}{\theta}\right] & \text{if } k_1 < 1 \\[2ex] -\kappa + \eta\dfrac{\partial v_1}{\partial k_1}(k'|y^*) & \text{if } k_1 > 1, \end{cases}$$

where $k' = k/(1+g) \in X(r, \tau - 1)$. By assumption $y_1^*(k') > 0$. Note that by the definition of y^*, this implies that $k_1' + y_1^*(k')$ is constant in a neighborhood of k' (that is, $y_1^*(k_1' + \epsilon, k_2') = y_1^*(k') - \epsilon$). Then, $\frac{\partial v_1}{\partial k_1}(k'|y^*) = m + \kappa$ for all such k' because a small increment $\epsilon > 0$ in k_1' increases the current profit by $m\epsilon$ and decreases the investment at the end of the period by ϵ. Therefore

$$\alpha(k) = \begin{cases} -\kappa + \eta[(m+\kappa) + m(1-\theta)/\theta] & \text{if } k_1 < 1 \\ -\kappa + \eta(m+\kappa) & \text{if } k_1 > 1. \end{cases} \tag{13}$$

Since $m/\kappa > (1-\beta)/\beta$, $\alpha(k) > 0$ if $k_1 < 1$. Equation (13) assumes that $y_1^*(k') > 0$, which allows us to compute $\frac{\partial v_1}{\partial k_1}(k'|y^*)$ explicitly. But even if $y_1^*(k') = 0$, we can compute $\frac{\partial v_1}{\partial k_1}(k'|y^*)$ (and hence $\alpha(k)$) recursively using the fact that in this case $\bar{r} > r$ and we already know $\hat{\tau}(j)$ for $j = 0, \ldots, r$.

We first check that if $\bar{r} \leq r$, so that firm 1 makes 'full investments' in I_r, then firm 1 also makes full investments in I_{r+1}. Let $\bar{\tau}$ be such that $(1+g)^{\bar{\tau}} \leq 1 + (1+g)^{r+1} < (1+g)^{\bar{\tau}+1}$. Assume that $\bar{r} \leq r$. Then $y^*(k') > 0$ for all $k' \in I_r$ with $k_1' < 1$ (in particular, for all $k' \in X(r, \bar{\tau} - 1) \cup X(r, \bar{\tau})$ with $k_1' < 1$). Let $k^1 \in X(r, \bar{\tau})$ with $k_1^1 < 1$, $k^2 \in X(r, \bar{\tau})$ with $1 < k_1^2 < 1 + g$ and $k^3 \in X(r, \bar{\tau} + 1)$ with $k_1^3 < 1 + g$. Since $y_1^*(k^1/(1+g)) > 0$, $\alpha(k^1) \geq 0$ by previous argument. By (12), it is easy to check that $\alpha(k^2)$ and $\alpha(k^3)$ have the same sign. Therefore, if $\alpha(k^2) < 0$, it is optimal for firm 1 to invest so that its final capacity is 1, and if $\alpha(k^2) > 0$ and $\alpha(k^3) > 0$, it is optimal for firm 1 to invest so that its final capacity is $1 + g$. Hence, let $\tau^*(r) = 0$ if $\alpha(k^2) < 0$ and let $\tau^*(r) = 1$ otherwise.

Now assume that $\bar{r} > r$, so firm 1 makes partial investments in I_r. In particular, $y^*(k) > 0$ for all $k \in X(r, \hat{\tau}(r) - 1)$ with $k_1 < (1+g)^{\tau^*(r)}$. Therefore $\alpha(k) > 0$ for all $k \in X(r+1, \hat{\tau}(r))$ with $k_1 < 1$. This implies that if firm 1 makes partial investments in I_{r+1} as well, then $\hat{\tau}(r+1) \geq \hat{\tau}(r) + 1$.

If $\alpha(k) \geq 0$ for all $k \in I_{r+1}$ with $k_1 < 1$ and $\alpha(k) < 0$ for all $k \in I_{r+1}$ with $k_1 > 1$, let $\bar{r} = r + 1$ and $\tau^*(r+1) = 0$. If $\alpha(k) \geq 0$ for all $k \in I_{r+1}$ with $k_1 < 1 + g$, let $\bar{r} = r + 1$ and $\tau^*(r+1) = 1$. If none of these two cases apply, then $\bar{r} > r + 1$, and we define

$$\hat{\tau}(r+1) = \max \{\tau \mid \alpha(k) \geq 0 \text{ for some } k \in X(r, \tau - 1)\},$$

and $\tau^*(r+1) = 0$ if $1 + (1+g)^{r+2} > (1+g)^{\hat{\tau}(r+1)}$ and $\tau^*(r+1) = 1$ otherwise. This finishes our analysis of the region I.

We now check that, for all $k \notin I$, $y_1^*(k)$ solves the optimization problem

$$\max_{y_1 \geq 0} \left[-\kappa y_1 + \beta \Big[\theta(1+g) v_1(k^+|y^*) + (1-\theta) v_1(k^0|y^*) \Big] \right],$$

where $k^0 = (k_1 + y_1, k_2 + y_2^*(k))$ and $k^+ = k^0/(1+g)$

Region L ∪ U: Let $k \in L \cup U$ and $y_1 < y_1^*(k)$. Then the derivative of the objective function of the optimization problem above is

$$\alpha = -\kappa + \beta\theta(1+g)\frac{\partial v_1}{\partial y_1}(k^+|y^*) + \beta(1-\theta)\frac{\partial v_1}{\partial y_1}(k^0|y^*).$$

Using (5) we obtain the lower bound

$$\alpha \geq -\kappa + \frac{\beta\theta}{2}\left[\frac{(2-\beta)m}{1-\beta} + \kappa\right] + \frac{\beta(1-\theta)}{2}\left[\frac{\beta m}{1-\beta} + \kappa\right]$$
$$= \frac{[\theta(2-\beta) + (1-\theta)\beta]\beta\kappa}{2(1-\beta)}\left[\frac{m}{\kappa} - \phi(\theta)\frac{1-\beta}{\beta}\right].$$

Since by assumption $\frac{m}{\kappa} \geq \phi(\theta)\frac{1-\beta}{\beta}$, we conclude $\alpha \geq 0$. Assume now that $y_1 > y_1^*(k)$. Here again, from (5) and (8) we obtain an upper bound on the derivative of the objective function:

$$\alpha \leq -\kappa + \frac{\beta\theta}{2}\left[\frac{\beta m}{1-\beta} + \kappa\right] + \beta(1-\theta)\frac{\eta}{2}\left[\frac{\beta m}{1-\beta} + \kappa\right] = \frac{\beta\eta\kappa}{2(1-\beta)}\left[\frac{m}{\kappa} - \frac{2-\eta}{\eta}\frac{1-\beta}{\beta}\right].$$

By assumption $\frac{m}{\kappa} \leq \psi(\theta)\frac{1-\beta}{\beta}$, and since $\psi(\theta)\frac{1-\beta}{\beta} < \frac{2-\eta}{\eta}\frac{1-\beta}{\beta}$, we conclude $\alpha < 0$. Therefore the objective function is (weakly) increasing for $y_1 \in [0, y_1^*(k))$ and (strictly) decreasing for $y_1 > y_1^*(k)$. Hence, $y_1 = y_1^*(k)$ is optimal.

Region $Z = \mathbf{R}^2 \backslash [L \cup U \cup I]$: Let $k \in Z$ and $y_1 \geq 0$. For either $k \in W$ or $k \in A$ it holds that $k_1^0 + k_2^0 \geq 1$ and $k_1^+ + k_2^+ \geq 1$ (to see why this is the case when $k \in W$, it suffices to recall that $y_2^*(k) = 1 + g - k_1 - k_2$). From (5) and (8), the derivative of the objective function has an upper bound:

$$\alpha \leq -\kappa + \beta\theta\frac{1}{2}\left[\frac{\beta m}{1-\beta} + \kappa\right] + \beta(1-\theta)\frac{\eta}{2}\left[\frac{\beta m}{1-\beta} + \kappa\right] < 0.$$

Now assume k is such that $k_1 \geq 1$. Since $\hat{\tau}(r) \geq 1$, the investment made by player 2 is such that $k_1^0 + k_2^0 \geq 1$ and $k_1^+ + k_2^+ \geq 1$. In this case, the best possible situation for firm 1 is when $k_1 < 1 + g$. In this case, an upper bound for the derivative of the objective function is obtained using (10):

$$\alpha \leq -\kappa + \beta\theta\frac{\beta m}{1-\beta} + \beta(1-\theta)\eta\frac{\beta m}{1-\beta} = -\kappa + \eta\frac{\beta m}{1-\beta} = \frac{\beta\eta\kappa}{1-\beta}\left[\frac{m}{\kappa} - \psi(\theta)\frac{1-\beta}{\beta}\right] \leq 0,$$

since by assumption $\frac{m}{\kappa} < \psi(\theta)\frac{1-\beta}{\beta}$. In all cases the objective function is weakly decreasing in y_1, and the optimal investment is $y_1 = 0 = y_1^*(k)$. ∎

7.2 *Proof of Theorem 2*

The proof follows the same arguments of the proof of Theorem 1. Let k be such that $k_1 + k_2 = 1$. With $\Delta = k_1 - k_2$ we can rewrite $k_1 = (1+\Delta)/2$ and $k_2 = (1-\Delta)/2$. In period 0, firm 1's payoff is $m(1+\Delta)/2$. In period 1, firm 1's expected payoff is

$$\theta\left[\frac{m}{2}(1+\Delta) - \kappa\frac{g}{2}\right] + (1-\theta)\frac{m}{2}(1+\Delta) = \frac{m}{2}(1+\Delta) - \kappa\frac{\theta g}{2}.$$

In general, in period $t \geq 2$, firm 1's expected payoff is:

$$\frac{m}{2}[(1+\theta g)^{t-1} + \Delta] - \kappa \frac{\theta g}{2}(1+\theta g)^{t-1}.$$

The expected discounted payoff is:

$$\begin{aligned} v_1(k|\hat{y}) &= \frac{m}{2}(1+\Delta) + \sum_{t=1}^{\infty} \frac{\beta^t}{2}[m[(1+\theta g)^{t-1} + \Delta] - \kappa[(1+\theta g)^{t-1}\theta g]] \\ &= \frac{m\Delta}{2(1-\beta)} + \frac{m(1-\theta g\beta) - \kappa\theta g\beta}{2(1-\gamma)} . \end{aligned}$$

If $k_1 + k_2 < 1$, let $O = 1 - (k_1 + k_2)$ and $\tilde{k}_i = k_i + O/2$, $i = 1, 2$. Here,

$$\begin{aligned} v_1(k|\hat{y}) &= v_1(\tilde{k}|\hat{y}) - m\tilde{k}_1 + mk_1 - \kappa\hat{y}_1(k) \\ &= v_1(\tilde{k}|\hat{y}) - m(k_1 + \frac{O}{2}) + mk_1 - \frac{\kappa}{2}(1 - k_1 - k_2) = v_1(\tilde{k}|\hat{y}) - (m+\kappa)\frac{O}{2}. \end{aligned}$$

The marginal value is therefore:

$$\frac{\partial v_1}{\partial k_1}(k|\hat{y}) = \frac{m}{2(1-\beta)} + \frac{m+\kappa}{2} = \frac{1}{2}\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right].$$

Let us now consider the case when k is such that $k_1 + k_2 > 1$ and $k_i < 1$. Suppose further that $(1+g)^\tau < k_1 + k_2 < (1+g)^{\tau+1}$, $\tau = 0, 1, 2....$ Let $k^\tau = (1+g)^{-\tau}k$. Using a similar argument as in the proof of Theorem 1 we obtain:

$$v_1(k|\hat{y}) = C(k_2) + [\eta(1+g)]^{\tau+1} v_1(k^\tau|\hat{y}).$$

So the derivative is:

$$\frac{\partial v_1}{\partial k_1}(k|\hat{y}) = \frac{\eta^{\tau+1}}{2}\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right].$$

We now check that $\hat{y}$ solves the following optimization problem:

$$\max_{y_1 \geq 0} \left[-\kappa y_1 + \beta[\theta(1+g)v_1(k^+|\hat{y}) + (1-\theta)v_1(k^0|\hat{y})]\right]$$

where $k^0 = (k_1 + y_1, k_2 + \hat{y}_2)$, $k^+ = k^0/(1+g)$. The derivative of the objective function is:

$$\alpha = -\kappa + \beta\theta(1+g)\frac{\partial v_1}{\partial y_1}(k^+|\hat{y})\frac{1}{1+g} + \beta(1-\theta)\frac{\partial v_1}{\partial y_1}(k^0|\hat{y}).$$

First Region: Assume that $k \geq 0$ and $k_1 + k_2 \leq 1$. If $y_1 < \hat{y}_1(k)$ then $k_1^0 + k_2^0 < 1$ and:

$$\alpha = -\kappa + \beta\theta\frac{1}{2}\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right] + \beta(1-\theta)\frac{1}{2}\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right] = -\kappa + \frac{1}{2}\beta\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right].$$

Thus, $\alpha \geq 0$ since by hypothesis $m/\kappa \geq (1-\beta)/\beta = \rho$, and firm 1 would like to increase y_1. Conversely, if $y_1 > \hat{y}_1(k)$ then $1 < k_1^0 + k_2^0$ and we obtain the following upper bound on the derivative:

$$\begin{aligned}\alpha &= -\kappa + \beta\theta(1+g)\frac{\partial v_1}{\partial y_1}(k^+|\hat{y}) + \beta(1-\theta)\frac{\partial v_1}{\partial y_1}(k^0|\hat{y}) \\ &\leq -\kappa + \beta\theta\frac{1}{2}\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right] + \beta(1-\theta)\frac{1}{2}\eta\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right] \\ &= -\kappa + \frac{\eta}{2}\left[\frac{m(2-\beta)}{1-\beta} + \kappa\right] \end{aligned} \tag{14}$$

Since by hypothesis $m/\kappa < (2-\eta)(1-\beta)/[\eta(2-\beta)]$, $\alpha < 0$ and firm 1 would like to decrease y_1. Therefore $y_1 = \hat{y}_1(k)$ is optimal.

Second Region: Assume that $k_1 + k_2 > 1$ and $k_i < 1$, $i = 1, 2$. In this case, if $y_1 > \hat{y}_1(k) = 0$ then $k_1^0 + k_2^0 > 1$ and the same upper bound (14) above attains. Therefore $\alpha < 0$ and firm 1 would like to decrease y_1.

Region $I = \cup_{r\geq 0} I_r$: Here, the construction of $\bar{r}$ and the maps $\tau^* : \mathbf{N} \to \{0,1\}$ and $\hat{\tau} : \{0, 1, \ldots, \bar{r}-1\} \to \mathbf{N}$ follows the same recursive steps leading to equation (12) in the proof of Theorem 1, but with a different boundary condition on the derivative $\alpha(k)$ for $k \in X(0,\tau)$ given by:

$$\alpha(k) = \begin{cases} -\kappa + \dfrac{\eta^\tau}{2}\left[\dfrac{m(2-\beta)}{1-\beta} + \kappa\right] + \eta m\dfrac{(1-\theta)}{\theta} & \text{if } k_1 < 1 \\ -\kappa + \dfrac{\eta^\tau}{2}\left[\dfrac{m(2-\beta)}{1-\beta} + \kappa\right] & \text{if } k_1 > 1. \end{cases}$$

The corresponding expression for $\alpha(k)$ in Theorem 1 given by (12) is more complex because of the two regions U and W required in the definition of y^*.

Third Region Assume that $k_1 \geq 1 > k_2$. Since $\hat{\tau}(r) \geq 1$, the investment made by player 2 is such that $k_1^0 + k_2^0 \geq 1$ and $k_1^+ + k_2^+ \geq 1$. In this case, the best possible situation for firm 1 is when $k_1 < 1+g$. In this case, an upper bound for the derivative of the objective function is given again by (14). Therefore $\alpha < 0$, the objective function is decreasing in y_1, and the optimal investment is $\hat{y}_1 = 0$.

7.3 *Proof of Theorem 3*

For any $k \in L$ this strategy generates the same stock trajectory as the strategy of Theorem 1. Therefore $v_1(k|y^*)$ satisfies equation (4) (with $\theta = 1$). In particular, if $k_1 + k_2 = 1$, then

$$v_1(k|y^*) = \frac{1}{2}\left[\frac{m-\kappa g}{1-\gamma} + \frac{m(k_1-k_2)}{1-\beta}\right] \quad \text{and} \quad v_1(k|y^*) + v_2(k|y^*) = \frac{m-\kappa g}{1-\gamma},$$

so $v_1(k|y^*) + v_2(k|y^*)$ is independent of k (as long as $k_1 + k_2 = 1$).

Let $k^0 \in M$. Then, for any $t \geq 0$,

$$k_i^t = \frac{k_i^0}{(1+g)^t} + \frac{g}{2}\sum_{\tau=0}^{t-1}\frac{1}{(1+g)^\tau} = \frac{k_i^0}{(1+g)^t} + \frac{1+g}{2} - \frac{1}{2(1+g)^{t-1}}.$$

When $k_i^0 = 1$, $k_i^t = \frac{1}{2}[\frac{1-g}{(1+g)^t} + 1 + g]$ for all $t \geq 0$. Hence,

$$v_1((1,1)|y^*) = \sum_{t\geq 0}\gamma^t\left[m(1-k_2^t) - \frac{\kappa g(1+g)}{2}\right] = \frac{1}{2}\left[\frac{m(1-g)-\kappa g(1+g)}{1-\gamma} - \frac{m(1-g)}{1-\beta}\right].$$

Moreover,

$$v_1((1-z,1-z)|y^*) = v_1((1,1)|y^*) + \frac{mz}{1-\beta} \quad \text{for all } 0 \leq z < \frac{1}{2}(1-g). \tag{15}$$

Clearly, $v_1((1-z,1-z)|y^*)$ is increasing in z, and for any $z < \beta g^2/[2(1-\gamma)] \equiv \epsilon_1$ and any k such that $k_1 + k_2 = 1$,

$$v_1((1-z,1-z)|y^*) < v_1((0,1)|y^*) \leq v_1(k|y^*). \tag{16}$$

We need to check the incentive constraints to follow y^*. We do this by regions.

First Region: Assume that $k^0 \in L$. In this case $v_1(k^0|y^*)$ satisfies equation (4). In Theorem 1 we checked that under-investments are not profitable when $k^0 \in L$. For similar reasons, under-investments are not profitable here. For over-investments we cannot use the results of Theorem 1 because when $k_1 + k_2 > 1$, the continuation values $v_1(k|y^*)$ are now different. Let $\{k^t\}$ be the capacity stock trajectory when the firms follow y^*. By definition, $k_1^1 + k_2^1 = 1$ and $k_i^1 \geq 1 - \bar{\epsilon}$, $i = 1, 2$. Suppose that firm 1 overinvests by $x(1+g)$ so that $(k_1^1 + x, k_2^1) \in A^\circ(0)$. We need to check that

$$\gamma v_1(k^1|y^*) \geq -\kappa x(1+g) + \gamma v_1((k_1^1 + x, k_2^1)|y^*).$$

At $(k_1^1 + x, k_2^1)$, firm 1 makes a revenue of $m(1-k_2^1)$ and needs to make an investment to bring its capacity stock up to $1 - \epsilon(k_1^1 + x, k_2^1) \geq 1 - \bar{\epsilon}$ in period 2. Recall that $v_1((1-z,1-z)|y^*)$ is increasing in z. Therefore

$$\begin{aligned}
&-\kappa x(1+g) + \gamma v_1((k_1^1 + x, k_2^1)|y^*) \\
\leq\ &-\kappa x(1+g) + \gamma[m(1-k_2^1) - \kappa((1-\bar{\epsilon})(1+g) - (k_1^1 + x))] + \gamma^2 v_1((1-\bar{\epsilon}, 1-\bar{\epsilon})|y^*) \\
<\ &\gamma[m(1-k_2^1) - \kappa((1-\bar{\epsilon})(1+g) - k_1^1)] + \gamma^2 v_1((1-\bar{\epsilon}, 1-\bar{\epsilon})|y^*) \\
<\ &\gamma[m(1-k_2^1) - \kappa g/2] + \gamma^2 v_1(k^2|y^*) = \gamma v_1(k^1|y^*),
\end{aligned}$$

where the last inequality follows because $k_1^2 \geq \bar{\epsilon}$ and (16) imply that $v_1((1-\bar{\epsilon}, 1-\bar{\epsilon})|y^*) \leq v_1(k^2|y^*)$, and $k_1^1 = 1 - k_2^1 \leq 1 - \bar{\epsilon}$ implies that $(1-\bar{\epsilon})(1+g) - k_1^1 \geq (1-\bar{\epsilon})g > g/2$. Thus an overinvestment of $x(1+g)$ with $0 < x < 1 + g - k_1^1$ is not profitable.

An overinvestment with $x \geq 1+g-k_1^1$ does even worse. In this case, $k_1^1+x \geq 1+g$ and $k_2^1 < 1$. Therefore, in period 1 the firms make no investments. Consider the overinvestment $\hat{x}(1+g)$ instead, where $k_1^1 + \hat{x} = (1+g)(1-\bar{\epsilon})$. By definition, $\hat{x} < x$, and one can also check that $v_1(1-\bar{\epsilon}, k_2^1) > v_1(k_1^1+x, k_2^1)$. Therefore, by the preceding argument,

$$-\kappa x(1+g) + \gamma v_1((k_1^1+x, k_2^1)|y^*) < -\kappa \hat{x}(1+g) + \gamma v_1(k_1^1+\hat{x}, k_2^1|y^*) < \gamma v_1(k^1|y^*).$$

Second Region: Assume $k^0 \in A^\circ(0)$ and $k_1^0 \neq k_2^0$. At the end of period 0, firm 1 is required to make an investment of $y_1^*(k^0) = (1-\epsilon(k^0))(1+g) - k_1^0$, so its total continuation value is

$$C = -\kappa y_1^*(k^0) + \gamma v_1((1-\epsilon(k^0), 1-\epsilon(k^0))|y^*).$$

Suppose that firm 1 overinvests $x(1+g)$ where $-y_1^*(k^0)/(1+g) < x < \epsilon(k^0)$ and $x \neq 0$, so that the capacity stock next period is $(1-\epsilon(k^0)+x, 1-\epsilon(k^0)) \in A^\circ(0)$. Then, its total continuation value is

$$-\kappa[y_1^*(k^0) + x(1+g)] + \gamma v_1((1-\epsilon(k^0)+x, 1-\epsilon(k^0))|y^*).$$

Overinvestments are clearly not profitable because they require an additional cost and lead to a lower continuation value from next period onward. The most attractive deviation is not to invest at all (that is, make $x = -y_1^*(k^0)/(1+g)$). Not investing leads to the capital stock $\hat{k}^1 = (k_1^0/(1+g), 1-\epsilon(k^0))$ and the total continuation value $\gamma v_1(\hat{k}^1|y^*)$. By construction, $\hat{k}^1 \in A(\bar{\epsilon}) \cup U(\bar{\epsilon}) \cup W(\bar{\epsilon})$. Therefore

$$\begin{aligned}\gamma v_1(\hat{k}^1|y^*) &= \gamma[m\epsilon(k^0) - \kappa((1-\epsilon(\hat{k}^1))(1+g) - \hat{k}_1^1)] + \gamma^2 v_1((1-\epsilon(\hat{k}^1), 1-\epsilon(\hat{k}^1))|y^*) \\ &\leq \gamma[m\bar{\epsilon} - \kappa((1-\bar{\epsilon})(1+g) - \hat{k}_1^1)] + \gamma^2 v_1((1-\bar{\epsilon}, 1-\bar{\epsilon})|y^*).\end{aligned}$$

On the other hand,

$$\begin{aligned}C &\geq -\kappa(1+g-k_1^0) + \gamma v_1((1-\epsilon(k^0), 1-\epsilon(k^0))|y^*) \\ &\geq -\kappa(1+g-k_1^0) + \gamma v_1((1,1)|y^*).\end{aligned}$$

To ensure that the firm does not want to deviate, we need to check that $C - v_1(\hat{k}^1|y^*) \geq 0$. The difference $C - v_1(\hat{k}^1|y^*)$ is bounded below by

$$\begin{aligned}&\kappa[\gamma((1-\bar{\epsilon})(1+g) - \hat{k}_1^1) - (1+g-k_1^0)] + \gamma(1-\gamma)v_1((1,1)|y^*) - \gamma m\bar{\epsilon}\left[1+\frac{\gamma}{1-\beta}\right] \\ &\geq \gamma(1-\gamma)v_1((1,1)|y^*) - (1-\gamma)\kappa(1+g) - \gamma\bar{\epsilon}\left[m\left[\frac{1-\beta g}{1-\beta}\right] + \kappa(1+g)\right],\end{aligned}$$

where equality is attained when $k_1^0 = 0 = \hat{k}_1^1$. Let

$$\varphi = \gamma v_1((1,1)|y^*) - \kappa(1+g) = \frac{\gamma m(1-g)}{2}\left[\frac{\beta g}{(1-\beta)(1-\gamma)}\right] - \kappa(1+g)\left[1+\frac{\gamma g}{2(1-\gamma)}\right].$$

Therefore $\varphi > 0$ if and only if

$$\frac{m}{\kappa} > \frac{(1+g)(1-\beta)}{\gamma\beta g(1-g)}[2(1-\beta) - \beta g(1-g)] = \rho\left[2\frac{\rho}{g(1-g)} - 1\right],$$

and the difference $C - v_1(\hat{k}^1|y^*)$ is nonnegative provided that

$$0 < \bar{\epsilon} \leq \frac{\varphi(1-\gamma)}{\gamma\left[m\left[\frac{1-\beta g}{1-\beta}\right] + \kappa(1+g)\right]} \equiv \epsilon_2.$$

Third Region: Assume that $k^0 = (1-z, 1-z) \in M$. If the firms follow y^*, then $k_1^1 = k_2^1 = \frac{1-z}{1+g} + \frac{g}{2}$. Any deviation leads to a $\hat{k}^1 \in A^\circ(0)$ with $\hat{k}_1^1 \neq \hat{k}_2^1$. One can check that the most attractive deviation is not to invest at all. Suppose firm 1 does not invest. Then $\hat{k}_1^1 = (1-z)/(1+g)$ and in the second period firm 1 is required to invest $(1-\bar{\epsilon})(1+g) - \hat{k}_1^1$. Hence, we need to check that

$$-\gamma\kappa\left[(1-\bar{\epsilon})(1+g) - \frac{1-z}{1+g}\right] + \gamma^2 v_1((1-\bar{\epsilon}, 1-\bar{\epsilon})|y^*) \leq -\kappa(1+g)\frac{g}{2} + \gamma v_1(k^1|y^*).$$

Since

$$\gamma v_1(k^1|y^*) - \gamma^2 v_1((1-\bar{\epsilon}, 1-\bar{\epsilon})|y^*) \geq \frac{\gamma m}{1-\beta}\left[\frac{z+g}{1+g} + \frac{g}{2} - \gamma\bar{\epsilon}\right]$$

and $z > 0$, the incentive constraint is satisfied if

$$\frac{\gamma m}{1-\beta}\left[\frac{g}{1+g} + \frac{g}{2} - \gamma\bar{\epsilon}\right] \geq \kappa(1+g)\frac{g}{2} - \gamma\kappa\left[(1-\bar{\epsilon})(1+g) - \frac{1}{1+g}\right]$$

or

$$\gamma\bar{\epsilon}\left[\frac{\gamma m}{1-\beta} + \kappa(1+g)\right] \leq \frac{\gamma m}{1-\beta}\left[\frac{g}{1+g} + \frac{g}{2}\right] - \kappa[(1+g)\frac{g}{2} - \beta((1+g)^2 - 1)].$$

Since $\beta > \frac{1}{2}$, $(1+g)g/2 - \beta((1+g)^2 - 1) < 0$. Hence, the incentive constraint is satisfied provided that

$$\bar{\epsilon} \leq \left[\frac{g}{1+g} + \frac{g}{2}\right] \Big/ \left[(1+g)(\beta + \frac{\kappa}{m}(1-\beta)\right] \equiv \epsilon_3.$$

Fourth Region: Assume that $k_i^0 > 1+g$, $i = 1, 2$. Firms are not supposed to invest. Any investment $x > 0$ undertaken by firm 1 such that $(1+g)^{\tau+1} > k_1^0 + x > (1+g)^\tau$ for some integer $\tau \geq 1$, has a net cost saving of:

$$[-1 + \beta^\tau]\kappa x < 0.$$

Therefore the optimal investment is $y_1^*(k^0) = 0$.

Fifth Region: Finally, assume that $k_1^0 < 1 + g \leq k_2^0$. Firm 1 is to invest $y_1^*(k^0) = 1 + g - k_1^0$ and firm 2 is to invest 0. For $y_1 < y_1^*(k^0)$, firm 1's marginal value of investment α is:

$$\alpha = -\kappa + \beta(m + \kappa) > 0.$$

Similarly, for $y_1 > y_1^*(k^0)$, the marginal value of investment is

$$\alpha \leq -\kappa + \beta\kappa < 0.$$

because an additional investment now produces no additional profits next period but decreases the required investment next period. Therefore the optimal investment is $y_1 = y_1^*(k^0)$.

In summary, if we choose $\bar{\epsilon} \leq \min\{\epsilon_1, \epsilon_2, \epsilon_3\}$, all incentive constraints are satisfied and y^* is an equilibrium. ∎

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