A Traffic Engineering Model for Air Taxi Services

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Abstract

In this paper, we propose a general model for on-demand air taxi services for a single service provider which unifies the previous research in this area. We then apply this model to the analysis of the week-to-week resource utilization of an air taxi service network. Towards that end, we develop two representations of the general air taxi service model to study several weekly utilization metrics at differing levels of abstraction. The first representation, a discrete-event model, records weekly levels of activity while specifying the minute-by-minute operations of a hypothetical air taxi service provider. The second representation, referred to as the flow model, is an aggregate model which describes the expected levels of activity for an air taxi service provider without specifying event-level operation. To "validate" the flow model against the more specific discrete-event model, we conduct a simple statistical comparison on the outputs of the two representations. Finally, we illustrate how the flow model can be applied towards the optimal pricing of passenger fares for a single air taxi service provider.

1 Introduction

The U.S. air transportation system is rapidly approaching saturation. In 2005, the system-wide load of the national airspace reached an all-time high of 77.1 percent, with 23 of the 35 top airports in the nation exceeding pre-9/11 activity levels. (FAA 2006) The FAA projects that up to 15 airports and 7 metropolitan areas will require additional capacity by 2013. (FAA 2004) With congestion and inefficiency throughout the air transportation system, the air passenger service industry has been exploring alternative strategies and technologies to existing scheduled, air carrier, hub-and-spoke service.

Accompanied by recent advances in small aircraft technology, one such alternative, on-demand air taxi service between rural, regional, and larger airports, has generated extensive interest. Although there is debate regarding how widespread the impact of air taxi services may be (Croft 2004), recent developments demonstrate that the potential of these services is already being realized. In some regions of the United States, SkyTaxi, Inc. (SkyTaxi 2005) has been providing on-demand air taxi service since 2002. More recently, the first class of very light jets (VLJs) is available to provide air passengers service to and from remote locations. (Miller 2006) Up to 10,000 VLJs are predicted to be in service by 2010. (Miller 2006) Meanwhile, ongoing efforts involve the development of a safe and affordable on-demand transportation service which not only increases the utilization of small airports and small passenger aircraft, but also reliably serves demand in a profitable fashion. (NASA 2004) The current research described herein focuses on the formulation of models for hypothetical air taxi service networks which can contribute to the development of effective operating principles for transportation planners.

In studies (RTI 2002, RTI 2004) of on-demand air taxi services, the Research Triangle Institute (RTI) uses large-scale Monte Carlo discrete-event simulations for North Carolina and the Great Plains states to demonstrate that a potential air taxi service could profitably accommodate a sizable daily demand at reasonable fares. However, the analysis by RTI assumes steady-state conditions and has scalability issues because it specifies minute-by-minute operation of a potential service network.

Alternative models for air taxi services can be constructed which both emphasize the evolution of the operation of a particular networked service and require less explicit description of the minute-by-minute activity of the service. One can look for inspiration in traffic engineering models, in application areas such as data networks (e.g. Elwalid et al. 2001, Low and Lapsley 1999) and road transportation (e.g. Wong et al. 2002), for models which capture the transient dynamics of pricing and allocation of service on a medium-range timescale. Motivated by traffic engineering models, Lee et al. (2005) develops a model for a hypothetical air taxi service network which generalizes more complicated events in the operation of the network by aggregating its activity on a weekly basis. In particular, Lee et al. (2005) consider a network with a fixed number of aircraft in which, on each route, the fare set by the service provider determines the rate and total of passenger arrivals for a week when the weekly number of revenue flights is fixed. The service provider can then use a gradient method to adjust its fare from week to week to improve the performance of the network (via the total profit) towards a stationary set of fares.

While these analyses provide promising results, their models describe highly specific hypothetical settings for air taxi services without defining a general structure for what constitutes an on-demand air taxi service. For example, in both RTI (2002) and RTI (2004), simulation models are constructed which characterize the demand for a potential on-demand air taxi service using the results of surveys of specific businesses in North Carolina and the Great Plains. Likewise, in Lee et al. (2005), a model is proposed for an air taxi service and studied for a simple three city scenario without more general applications of the model. Thus, this effort defines a general model for on-demand air taxi services, which one can use to model specific scenarios as a discrete-event simulation model, a model like that in Lee et al. (2005), or some other type of model.

In this paper, we first propose a general model for on-demand air taxi services for a single service provider. The goal of this paper is to use the framework of the general service model to instantiate more specific representations for analyzing and comparing the resource utilization of a single air taxi service provider. For the purpose of studying the week-to-week utilization of a hypothetical air taxi service, we then develop a discrete-event model derived from the general service model. In particular, we focus on the levels of passenger demand, revenue flights, and deadhead flights, which are the base utilization metrics we can use to compute revenues and costs. In addition, we address the issue of model fidelity by formulating a traffic engineering flow model for an air taxi service network which extends the model of Lee et al. (2005). We wish to use this model to capture, in aggregate form, weekly utilization outputs of an air taxi service similar to those of a discrete-event model. If we are concerned that the level of specificity necessary for a discrete-event model becomes unwieldy for a large-scale problem, we may wish to use some higher-level model such as the flow model. What then becomes a concern is evaluating whether a less specific model, like the flow model, really allows us to produce similar basic utilization metrics as a discrete-event model.

Finally, with a longer timescale to analyze the operation of an air taxi service, the flow model can then be used to make decisions about how to adjust fares for using the service. In particular, an air taxi service provider will generally be interested in how to set fares for passengers in order to maximize or minimize some performance measure, be it total profit, cost of deadheading, or some other measure. Given the complex and dynamic nature of the service provider's optimization problem, an appropriate iterative procedure would be necessary to arrive at a solution. Thus, the last contribution of the work is an application of the flow model towards optimizing the profit for an air taxi service by adjusting fares for the service using a gradient-based method from nonlinear programming.

The remainder of this paper proceeds as follows: Section 2 describes our general air taxi service model, emphasizing how it both fits into the existing research on on-demand air transportation and generalizes the concept of an air taxi. We also discuss the development of a discrete-event model, based on the general service model, for investigating changes in air taxi service resource utilization on a week-to-week basis. In Section 3, we provide a general definition of the flow model and a description of the development of a comparison scenario to the discrete-event model. Section 4 presents preliminary results of a comparison of the outputs from the discrete-event simulation and flow models. Section 5 illustrates an application of the flow model using a gradient-based pricing algorithm. Section 6 concludes this paper and discusses future directions for the work.

2 A Single-Provider Air Taxi Service Model

In this section, we describe a general air taxi service model for a single service provider, which unifies the prior research conducted in this and related areas. We begin in Section 2.1 with a review of the literature for on-demand air transportation and follow, in Section 2.2, with a description of our service model in the context of the existing research. Finally, for the purpose of addressing pricing and provisioning applications over medium-range (i.e., weekly) timescales, we develop a discrete-event model which describes the utilization of air taxi resources as a function of price-modulated customer arrival rates, specific dispatch strategies, and other relevant factors over a one-week operation period. We focus our analysis on the basic utilization metrics of the service network, like the passenger load and revenue and deadhead flight levels, from which we can derive other significant metrics like costs and revenues.

2.1 Prior Studies of On-Demand Air Transportation Services

The current state-of-the-art in the modeling of on-demand air transportation lacks a consensus. As in RTI (2002) and RTI (2004), Bonnefoy (2005) presents a simulation study of the long-term operations of a large-scale air taxi network and the effects of design variables such as fleet size, demand distribution and network size. The study additionally emphasizes the complexity and unstructured, evolving nature of air taxi networks and suggests that aggregate models, as in Lee et al. (2005), may not characterize such networks accurately.

Another take on air taxi service modeling, as a standard scheduling optimization problem, can be found in Cordeau et al. (2004). This study presents both static and dynamic versions of the *dial-a-flight problem*, in which an air taxi service provider schedules a set of passenger flight requests for one day. In the static version, the provider optimizes profit and plans crew and aircraft itineraries to satisfy all requests subject to constraints on each request's departure time as well as aircraft and crew availability. In the dynamic version, the provider must choose to accept or reject requests as they arrive, subject to similar availability constraints (as in the static problem) and the other requests already accepted.

A related form of on-demand transportation, fractional aircraft ownership (Foster 2001, Levere 1996, Zagorin 1999), has also garnered increasing interest. Fractional ownership programs guarantee users on-time flight service whenever and wherever necessary on short notice as part of an agreement with a fractional management company (FMC). An FMC allows a user to purchase part of an aircraft based on the number of flight hours needed in a given year. A so-called fractional owner then requests a flight by calling the FMC and specifying a desired origin and destination and departure time. The FMC must then assign an aircraft to serve the request, by having one already available at the requested origin, by deadheading one of its own aircraft, or by subcontracting an external aircraft. For a collection of distinct requests in a one or two day period, the FMC determines a cost-minimizing assignment of flight sequences to aircraft and pilot crews subject to constraints on pilot availability and maintenance schedules.

Previous work (see Karaesman et al. 2003, Keskinocak and Tayur 1998, Martin et al. 2003, Ronen 2000, Yao et al. 2005) on fractional ownership aircraft service considers variants on the static problem of scheduling fractional ownership aircraft over a fixed scheduling horizon. Both Keskinocak and Tayur (1998) and Ronen (2000) address the problem of scheduling aircraft, though only the latter considers crew availability constraints, subcontracting of aircraft to meet requests, and multiple horizon lengths. As a complement to Keskinocak and Tayur (1998) and Ronen (2000), the work in Karaesman et al. (2003) develops several models and methods for the combined scheduling of crew duties and multiple types of aircraft for a single day of operations. These models and methods are tested on both randomly-generated and real data to find optimal and near-optimal solutions. Using some of the modeling elements in Karaesman et al. (2003), Martin et al. (2003) describes the development of a decision support tool used by Raytheon Travel Air to assist in its combined crew and aircraft scheduling decisions. Finally, Yao et al. (2005) builds upon these other works by considering flexibility in departure times and a rolling scheduling horizon to deal with fluctuations in demand.

2.2 Description of the Service Model

The models in Lee et al. (2005), RTI (2002), RTI (2004), and to a lesser extent, Bonnefoy (2005), treat an air taxi service similarly to a traditional passenger flight service, in which service is provided on a fixed network of cities. Our service model also views an air taxi service in this fashion, although Cordeau et al. (2004) and the fractional ownership literature do not include this restriction. We do not view an air taxi service as a concept similar to the fractional ownership industry, i.e., as an essentially exclusive commodity in which the user dictates most of the terms of service. We do, however, consider key aspects of the fractional ownership aircraft scheduling problems: the aircraft owned by the service provider can sit idle at any city in the network and are not constrained to return to a "home base" (as in Cordeau et al. 2004), the aircraft can be deadheaded, and the aircraft are of more than one type. We adopt these characteristics to capture the on-demand nature needed for modeling air taxi services.

Service Structure

Our general air taxi service model specifies that a service provider operates single-leg, point-to-point service on a fixed network of cities, of any arbitrary size and coverage. This service utilizes a fixed and finite number of aircraft which are classified by both different passenger capacities and cruise velocities, although it is possible for different aircraft types to be identical in these characteristics. The total number of aircraft must be greater than one, although there does not have to be at least one aircraft ready to fly each individual route. Thus, aircraft can fly any route in the network without having to cycle to and from a "home base." Aircraft can be deadheaded when necessary, and, as in Bonnefoy (2005), Karaesman et al. (2003) and Martin et al. (2003), both deadhead and revenue flights incur a fixed operating cost for each flight hour. Additionally, the typical FAA regulations such as those on crew availability and required maintenance scheduling for aircraft apply in the general service model.

Customer-Provider Interaction

We characterize how passengers request service by looking towards the dynamic version of the *dial-a-flight problem* as described in Cordeau et al. (2004). However, instead of handling the arrivals of flight requests from passengers, it is the passengers themselves who arrive to fly a specific route *after* having requested service some short period of time in advance (e.g., for most FMCs it is three to four days (Keskinocak and Tayur 1998, Martin et al. 2003)). Each passenger who requests service is expected to show up for his/her flight. As in both RTI (2002) and RTI (2004), but not in any fractional aircraft ownership models, it is possible for passengers to arrive with no lead time (i.e., to walk in exactly when they wish to fly, request service and then fly). All arriving passengers pay a fare, specific to each route, which is held constant for at least some medium-range time horizon (e.g., one week). The fare is the same for all passengers, regardless of when they request service.

As with the provider's accept/reject decision with respect to passenger requests in Bonnefoy (2005) and Cordeau et al. (2004), the provider can deny service to an arriving passenger who has already requested service. Denial of service is restricted only to those arriving passengers whose request is made beyond some cutoff lead time, set very shortly in advance of departure (such as less than 24 hours in advance). Service denials will occur between the time of a potential passenger's request and arrival, and occur due to capacity restrictions for the next departing flight on the requested route. A denied passenger is then reimbursed the fare paid for the flight and is awarded some reward, such as credit toward a future flight. Passengers who are not turned away under such circumstances are assigned to the next available flight. These passengers, who make their requests before the cutoff lead time, are also able to specify the maximum amount of time they are willing to wait for their flight, similar to the earliest and latest departure times passengers can specify in the models in Bonnefoy (2005) and Cordeau et al. (2004).

2.2.1 Discussion

Our air taxi service model is general in the sense that it is not limited to a single problem type and is independent of any specific assumptions regarding the scale and scope of the service. This model describes an air taxi service as a combination of a fixed service network, fluid aircraft assignment, and strong passenger-provider interaction which have been part of the different modeling efforts for on-demand air transportation. Thus, the model represents an integration of the concept of an on-demand flight service as mass transit, as seen in Bonnefoy (2005), RTI (2002) and RTI (2004), and the more "personalized" models seen in Cordeau et al. (2004), Karaesman et al. (2003), Keskinocak and Tayur (1998), and Ronen (2000).

Additionally, the air taxi service model presented here allows for flexibility in application towards solving a variety of problems in on-demand air transportation. Previous modeling efforts have focused on the short-term scheduling of a batch of flight requests or the economic viability of the long-term operation of a future air taxi service, and the definition of variables and parameters have been tailored to these specific problems. The model we propose defines the basic characteristics of the type of service provided and what a potential passenger does, without detailing problem-specific features like sets of feasible schedules and probability distributions on demand.

Finally, the model is not specific to any single example air taxi service network. We avoid limiting the size and coverage of an air taxi service, though previous work (as in RTI 2002, RTI 2004) has emphasized air taxis as serving business passengers for regional transportation purposes.

2.3 A Discrete-Event Representation

Having presented a general service model for future air taxi services, we now seek to understand the effect of the service model on a single service provider's utilization of resources. We are particularly interested in revealing medium-horizon economic tradeoffs relating to the pricing of services and provisioning of aircraft, on say a weekly time scale. As a reference for characterizing the basic week-to-week operations of a single service provider, we develop a discrete-event simulation model that accounts for operational variables including passenger arrivals, revenue flights, deadhead flights, service denials, service denial penalties and overall aircraft utilization during the given time horizon. Since we envision an air taxi service operating more like a traditional airline than an FMC, long-range operational decisions like pricing and provisioning need to be considered. To better inform these decisions, we thus use our discrete-event representation to study the average cumulative levels of service for a potential air taxi service over longer time scales. In this initial formulation, the discrete-event model neglects the effects of weather, cancellations, and other airspace issues. We also do not focus upon other details of typical air transportation operations such as crew and maintenance scheduling.

In the discrete-event model, every flight requires some amount of ground time, before takeoff and after landing, for standard procedures such as boarding, clearing, waiting for pushback, taxiing, and deplaning. The duration of this standard ground time is a non-negative random variable. Similarly, the duration of any delay for any individual flight, both in-transit or on-ground, is also a non-negative random variable, whose distribution is different from those for each ground time duration. Unlike the air taxi service model in Cordeau et al.

(2004) and the fractional aircraft ownership literature, the service provider's objective is, subject to constraints on passenger capacity, to determine a profit-maximizing assignment of aircraft and set of route prices. The provider's optimization problem changes because passenger requests are not necessarily fully satisfied and prices affect whether these service denials occur.

Input Parameters and Variables

More formally, the discrete-event model specifies that an air taxi service consists of:

- V, the set of cities in the service network, for which a route (i, j) exists between distinct cities $i, j \in V$ and has:
  - distance d_{ij} and
  - weekly fare P_{ij}, and
- AC^k, the number of aircraft of type $k \in A$ which have:
  - passenger capacity Q^k,
  - average cruise velocity v^k, and
  - for each route (i, j), a flight duration $T_{ij}^k = v^k d_{ij}$.

The total number of aircraft is

$$AC = \sum_k AC^k \geq 2. \tag{1}$$

Assuming 24-hour service and no maintenance or other down-time, the maximum number of flight hours for the AC aircraft is the constant

$$MFH = \sum_k \sum_{(i,j)} AC^k \frac{168}{T_{ij}^k} \tag{2}$$

Arriving passengers are classified into:

- Class 1, passengers requesting service more than one day in advance who are each associated with:
  - a unique maximum waiting time W_{max} a non-negative random variable with the cumulative distribution function $F_{W_{max}}$, and
- Class 2, the remaining passengers who are denied service when they arrive to route (i, j) for a flight on a type k aircraft and find Q^k passengers already waiting.

The service provider incurs two main costs:

- CO, a fixed, per-flight hour operating cost, and
- H_{ij}, a weekly variable penalty cost to compensate denied passengers.

Finally, the discrete-event model specifies two input variables which affect service performance:

- G_d, the delay duration for any individual flight, a non-negative variable with cumulative distribution function F_{G_d}, and
- G, the duration of standard ground time for any individual flight, a non-negative random variable with cumulative distribution function F_G.

Output Variables

The discrete-event model specifies the following counters:

- D_{ij}, the weekly passenger arrivals on (i, j),
- S_{ij}, the weekly number of revenue flights on (i, j),
- DH_{ij}, the weekly number of deadhead flights on (i, j),
- O_{ij}, the weekly number of passenger service denials on (i, j), and

- FH_{ij}, the weekly total flight hours on (i, j).

These counters can be viewed as the tabulation of the main outputs of the air taxi service model. We can use these counters to derive other relevant output variables:

- U, the weekly overall utilization (percentage of actual flight hours to maximum possible flight hours) of the AC aircraft,
- REV_{ij}, the weekly revenue on (i, j),
- FC_{ij}, the weekly cost of revenue flights on (i, j),
- DC_{ij}, the weekly cost of deadhead flights on (i, j),
- OC_{ij}, the weekly penalty cost of service denials on (i, j),
- C_{ij}, the total weekly cost incurred to the service provider on (i, j), and
- Z_{ij}, the total weekly profit from (i, j).

2.3.1 An Illustrative Example

We now describe an example scenario for the discrete-event model for the simple case with only one type of aircraft and Class 1 passengers requesting the service. This discrete-event model includes the additional assumption that passengers arrive to routes according to a non-homogeneous Poisson Process, after having requested service in advance. These passengers are all assumed to be Class 1 passengers, who will not be denied service and each specify that $W_{max} = 1$ hour. We also assume that the ground time and delay variables are all degenerate random variables which equal zero with probability one.

The service provider operates a simple network consisting of three equidistant cities (60 nm apart) served by two aircraft. A single-leg route connects each pair of cities (in each direction), each of which is available for passenger travel. The utilized aircraft are assumed to carry up to 4 passengers at an average cruise velocity of 300 knots. The provider incurs a cost on each flight at $1100 per flight hour and gains revenue at $2.00 per passenger mile. Passengers arrive to each city according to a non-homogeneous Poisson Process, and then travel on each outgoing route from that city with equal probability. We run this scenario over the following range of values for the arrival rate μ_i to each city $i \in V$ for any week: $\{1.00, 1.11, 1.25, 1.43, 1.67, 2.00, 2.50, 3.33, 5.00\}$. These arrival rates are derived from incrementing passenger interarrival times to each city by 0.10 hour on a range from 0.20 to 1.00 hours. We use these arrival rates to investigate the average behavior of the network for a range of potential passenger demands from moderately low (0.5 passengers/hr) to moderately high (2.5 passengers/hr) to each route.

The above scenario is implemented, for each arrival rate, using the discrete-event simulation package ARENA (version 9.0) (Rockwell Software 2004). The three cities in the service network are numbered arbitrarily. Tables 1, 2, and 3 show weekly averages and standard deviations on all routes (over 1000 simulation runs) for passenger arrivals, revenue flights, and deadhead flights for each arrival rate in our specified range. Table 4 shows the same results for overall aircraft utilization. Table 5 shows the results of an ANOVA which tests, for each specified arrival rate, the equality across routes of the average number of passenger arrivals, revenue flights, and deadhead flights. At a significance level of 0.05, the results indicate that there are no significant differences between routes for the average number of passenger arrivals and revenue flights, but the same is not true for the average number of deadhead flights. A more detailed description of the implementation of this discrete-event model can be found in Boyd et al. (2006).

3 Flow Model

In this section, we extend the model proposed in Lee et al. (2005) to what we now refer to as the flow model. This model describes the utilization of the service provider's network on a longer timescale than the actual minute-by-minute operation of the system. The main benefit (and technical challenge) of the flow model is that it does not account for specific events in the actual operation of the system, such as individual passenger arrivals, revenue flights, or deadhead flights, as they actually occur in the network being modeled. The flow model instead uses mathematical functions to capture the aggregate occurrence of these events in an air taxi service over the course of a week. Specifically, the weekly fares for each route in the service still determine the rate and total passenger arrivals for the week. However, the functions for passenger arrivals to each route in the service are parameterized more in the context of the service model defined in Section 2. In addition, the expected weekly number of revenue flights on each route, rather than being fixed, is now estimated as a function of the fare-dependent number of passenger arrivals for the week. We also extend the model of Lee et al. (2005) to consider deadheading of aircraft, where the expected weekly number of deadhead flights on a route is estimated as a function of the both the expected weekly number of revenue flights on that route and the total number of aircraft utilized by the network. These model specifications are all made in the context of the discrete-event model presented in Section 2.3, with the one distinction that passengers can be denied service.

3.1 Description

Following the definition of the air taxi service model in Section 2, suppose that for the route (i, j) for $i, j \in V$, the weekly fare is P_{ij}. Introducing now a constant maximum demand parameter B_{ij} for the route (i, j) for any week, the weekly demand (number of passenger arrivals) on (i, j) is defined as

$$D_{ij}(P_{ij}) = B_{ij}e^{-0.01P_{ij}}, \tag{3}$$

so that the service provider earns the following weekly revenue from (i, j):

$$REV_{ij}(P_{ij}) = P_{ij}D_{ij}(P_{ij}). \tag{4}$$

The particular form of (3) is chosen so that demand is a bounded, convex and smooth function of the fare P_{ij}, which decays exponentially. The exponential decay, however, is not too rapid, so that for well-selected values of B_{ij} (see Section 3.2.1) weekly passenger arrivals will still be in the hundreds when the fare well exceeds \$ 100..

The weekly demand (passenger arrivals) given in (3) to (i, j), determines the weekly expected number of revenue flights $S_{ij}(D_{ij}(P_{ij}))$ needed to accommodate the arriving passengers for that week. We now introduce the passenger arrival rate per individual flight (as a function of the weekly fare for (i, j), P_{ij}) $\lambda_{ij}(P_{ij})$ to (i, j). Assuming that the expected number of revenue flights $S_{ij}(D_{ij}(P_{ij}))$ are evenly spaced throughout the week, the passengers arriving to (i, j) for any single flight is the Poisson Process $M_{ij}(t)$ where

$$\lambda_{ij}(P_{ij})t = \frac{D_{ij}(P_{ij})}{S_{ij}(D_{ij}(P_{ij}))}t. \tag{5}$$

So the number of weekly passenger arrivals $M_{ij}(t)$ for each flight on (i, j) is a Poisson random variable with mean $\lambda_{ij}(P_{ij})t$, where $t = 1$. Thus,

$$P(M_{ij}(1) = k) = e^{-\lambda_{ij}(P_{ij})}\frac{\lambda_{ij}(P_{ij})^k}{k!}. \tag{6}$$

Characterizing the passenger arrival process for each individual flight as a Poisson Process allows for an associated definition of passenger service denials FO_{ij} for any single flight on (i, j) during the week (i.e., the number of passengers arriving over capacity for the flight). Assuming the capacity for any flight is some constant Q, the expected number of passenger service denials on (i, j) for any single flight during the week is

$$E[FO_{ij}] = \sum_{k=Q+1}^{\infty} (k - Q)P(M_{ij}(1) = k) \tag{7}$$

Since $\sum_{k=0}^{\infty} kP(M_{ij}(1) = k) = E[M_{ij}(1)] = \lambda_{ij}(P_{ij})$ and $\sum_{k=0}^{\infty} P(M_{ij}(1) = k) = 1$, (7) can be rewritten as

$$E[FO_{ij}] = \left(\lambda_{ij}(P_{ij}) - \sum_{k=0}^{Q} kP(M_{ij}(1) = k)\right) - \left(1 - \sum_{k=0}^{Q} P(M_{ij}(1) = k)\right) \tag{8}$$

Since (8) gives the expected passenger service denials on (i, j) for a single flight during the week, and applying the stationary and independent increments property of the Poisson Process, the expected number of passenger service denials for the entire week is

$$E[O_{ij}(P_{ij})] = S_{ij}(D_{ij}(P_{ij}))E[FO_{ij}]. \tag{9}$$

Defining H_{ij} as a penalty cost the service provider must pay for each passenger service denials on (i, j) for the week, the expected overage cost is computed as

$$E[OC_{ij}(P_{ij})] = H_{ij} \cdot E[O_{ij}(P_{ij})]. \tag{10}$$

Since some revenue flights require the deadheading of an aircraft from a city other than the origin, there is a weekly deadhead cost $E[DC_{ij}]$ for each route (i, j). Given the expected weekly number of deadhead flights (as a function of the expected weekly number of revenue flights and the number of aircraft) $DH_{ij}(S_{ij}(D_{ij}(P_{ij})), AC)$ on (i, j), the average flight duration on (i, j) T_{ij}, and the fixed per-flight hour operating cost CO, the expected weekly deadhead cost for (i, j) is

$$E[DC_{ij}(P_{ij})] = CO \cdot T_{ij} \cdot DH_{ij}(S_{ij}(D_{ij}(P_{ij})), AC) \tag{11}$$

The final cost component is the expected weekly cost of revenue flights FC_{ij} given as

$$FC_{ij}(P_{ij}) = CO \cdot T_{ij} \cdot S_{ij}(P_{ij}) \tag{12}$$

So the total expected weekly cost for route (i, j) is then computed as

$$E[C_{ij}(P_{ij})] = FC_{ij} + E[DC_{ij}(P_{ij})] + E[OC_{ij}(P_{ij})] \tag{13}$$

giving the expected weekly profit for (i, j) as

$$E[Z_{ij}(P_{ij})] = REV_{ij}(P_{ij}) - E[C_{ij}(P_{ij})] \tag{14}$$

and the total expected weekly profit for the entire network as

$$E[Z(P)] = \sum_{i,j \in V} E[Z_{ij}(P_{ij})] \tag{15}$$

where P is the vector of the fares charged to all routes (i, j) during the week.
To compute the overall aircraft utilization of the network, we first compute the total flight hours for the week on (i, j) as

$$FH_{ij} = T_{ij}(S_{ij}(D_{ij}(P_{ij})) + DH_{ij}(S_{ij}(D_{ij}(P_{ij})), AC)) \tag{16}$$

the product of the flight duration on the route and the sum of the expected weekly levels of revenue and deadhead flights on the route. The total flight hours for the AC aircraft is then just the sum of FH_{ij} over all routes (i, j) for the week. The ratio of this value to (2) is the weekly overall aircraft utilization U of the AC aircraft.

3.2 Model Identification

We now turn our attention to extending the model presented in Lee et al. (2005) by (1) providing a method for parameterizing the "maximum demand" parameter, (2) determining functions for both the weekly expected number of revenue and deadhead flights on each route, and (3) defining the "penalty costs" for service denial on a route in terms of the route fare. In addition to describing the specification of these parameters and variables in general cases, we apply these specification procedures to the example scenario for the discrete-event model in Section 2.3.1.

3.2.1 Maximum Demand

Assuming some stochastic characterization for the arrival of passenger demand, we describe the maximum demand B_{ij} on route (i,j) for any week (i.e., in view of (3), the number of passengers arriving to (i,j) during any week when the fare is zero) as a demand level which is exceeded with a small probability σ. For example, in the illustrative example for the discrete-event model in Section 2.3.1, we assume that passengers arrive to each city i according to a non-homogeneous Poisson Process and then move on to each outgoing route (i,j) with equal probability. For the three-city case in Section 2.3.1, the maximum number of passengers arriving over any hour is the first integer K such that

$$P(\geq K) = \sum_{k=0}^{K} e^{-\mu_i/2} \frac{(\mu_i/2)^k}{k!} \geq 1 - \sigma \tag{17}$$

Solving the above for the arrival rates listed in Section 2.3.1 and $\sigma = .01$ gives $4 \leq K \leq 7$ for one hour, so for the entire week, the nominal "maximum demand" is in [672, 1176]. In order to use the flow model for an example scenario analogous to that in Section 2.3.1, we use the "maximum demand" parameter values given in Table 6. These values are varied in the upper range of possible "maximum demand" values, so that the routes are neither completely identical to one another nor sufficiently different from one another.

3.2.2 Expected Number of Revenue Flights

In section 3.1, the expected weekly number of revenue flights $S_{ij}(D_{ij}(P_{ij}))$ needed to accommodate the weekly demand $D_{ij}(P_{ij})$ on (i,j) was introduced without specification of any functional form. The desired form of $S_{ij}(D_{ij}(P_{ij}))$ needs to meet the following requirements:

- $S_{ij}(D_{ij}(P_{ij})) = 0$ when $D_{ij}(P_{ij}) = 0$,
- $S_{ij}(D_{ij}(P_{ij}))$ is monotonically increasing in $D_{ij}(P_{ij})$,
- $S_{ij}(D_{ij}(P_{ij}))$ is bounded by and approaches some finite value as $D_{ij}(P_{ij}) \to \infty$.

The first requirement imposes the obvious condition that no revenue flights are scheduled if there is zero demand, while the second and third requirement ensure that more revenue flights will accommodate more demand but only up to some physical limit, since we assume, for practical purposes, a finite number of aircraft in service. One such functional form meeting these requirements is a power function

$$S_{ij}(D_{ij}(P_{ij})) = A_{1_{ij}} \cdot [D_{ij}(P_{ij})]^{A_{2_{ij}}} \tag{18}$$

in which $A_{1_{ij}}$ and $A_{2_{ij}}$ are parameters to be estimated, for each route (i, j), using regression procedures.

For the example presented in Section 2.3.1, estimation of (18) is performed with flight and demand data from, respectively, the average passenger arrivals and average number of revenue flights resulting from a number of experiments of the discrete-event simulation scenario as presented in Section 2.3.1. These data are taken for a single arbitrary route, as the example assumes routes of equal distance and demand arrival rate. Furthermore, the data from Section 2.3.1 exhibit no statistically significant differences in expected number of revenue flights for all routes.

A scatter of the log-log transformation of the average number of revenue flights versus the average passenger arrivals from the simulation follows in Figure 1. Data which should be fit with a power function will show a linear dependence in a log-log transformation (Neter et al. 1996), and the log-log plot of the average number of revenue flights versus average passenger arrivals suggests a linear relationship (slope = 0.667, intercept = 1.016, $p < .001$). Using

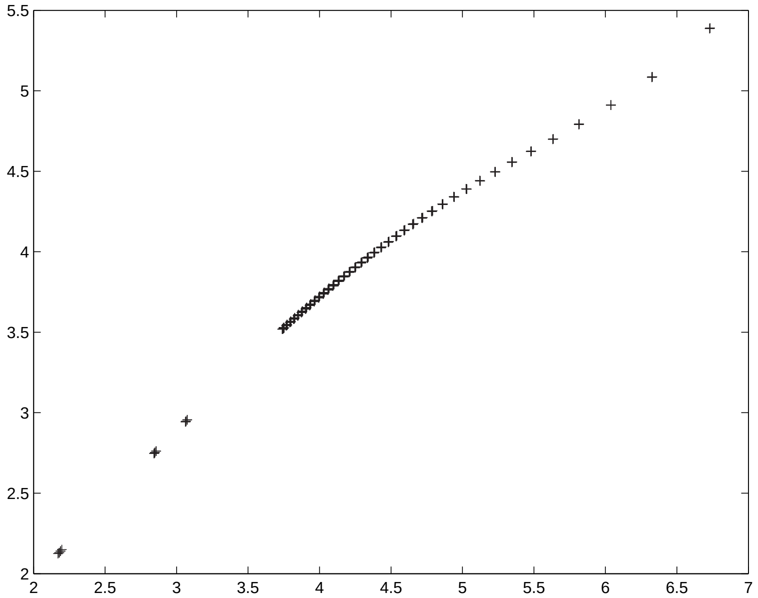

Figure 1: Log-Log scatter plot of weekly average revenue flights vs. weekly average passenger arrivals

the nonlinear regression routine nls in S-PLUS (Version 6.2) (Insightful 2004), the model

$$S_{ij}(D_{ij}(P_{ij})) = 3.867 \cdot [D_{ij}(P_{ij})]^{0.5964} \tag{19}$$

is approximately estimated, for all routes (i, j) with a standard error of 1951.78 and $R^2 = .9950$. Figure 2 provides a scatter of the untransformed data with the corresponding fitted values of (19).

3.2.3 Expected Number of Deadhead Flights

The weekly number of deadhead flights on (i, j), $DH_{ij}(S_{ij}(D_{ij}(P_{ij})), AC)$ was previously described as a function of the weekly expected number of revenue flights $S_{ij}(D_{ij}(P_{ij}))$ on (i, j) and the total number of aircraft AC without any functional form. Figure 3 displays a scatter of the average weekly deadhead flights versus the average weekly revenue flights on an individual route when the service provider has two through four aircraft available for use. The data are averages over 1000 runs of the discrete-event example described in Section 2.3.1. For each aircraft level, a similar unimodal shape is seen with deadhead flights

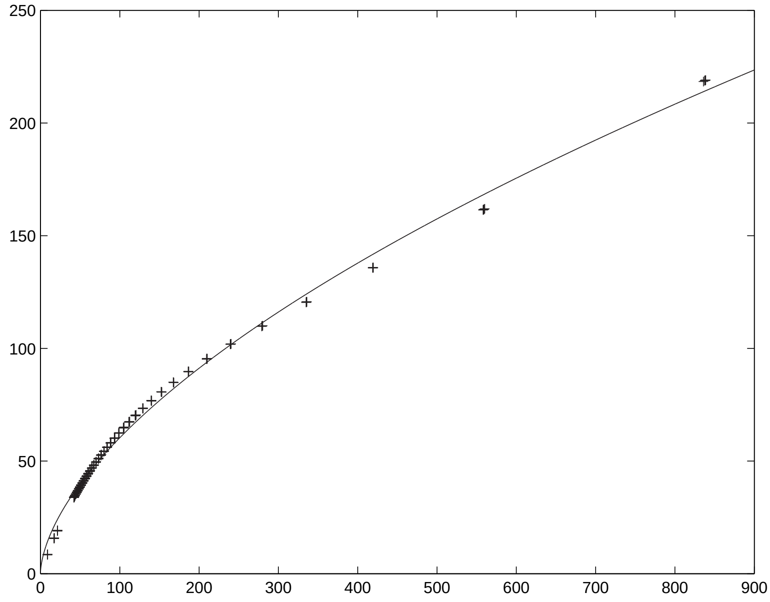

Figure 2: Scatter plot of weekly average revenue flights versus weekly average passenger arrivals with the fit (19)

increasing with revenue flights, followed by a rapid decrease until deadheading approaches zero. There is, however, variation in the shape of each scatter as the aircraft level increases– the peak deadhead level increases and the concave downward portion of the scatter widens. To specify a functional form, we consider several families of probability distributions whose shapes vary with specification of some shape parameter and which exhibit a similar shape to the scatter plots seen in Figure 3. One such family is the gamma family of distributions, which has the probability density function

$$\frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta} \tag{20}$$

where α is a shape parameter, β is a scale parameter and $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$. We propose a similar form for the weekly expected number of deadhead flights on any route (i, j), as a function of the weekly expected number of revenue flights on the route and the number of aircraft in operation:

$$DH_{ij}(S_{ij}(D_{ij}(P_{ij})), AC) = A_{3_{ij}} \cdot (S_{ij}(D_{ij}(P_{ij})))^{AC} \cdot e^{-A_{4_{ij}} S_{ij}(D_{ij}(P_{ij}))} \tag{21}$$

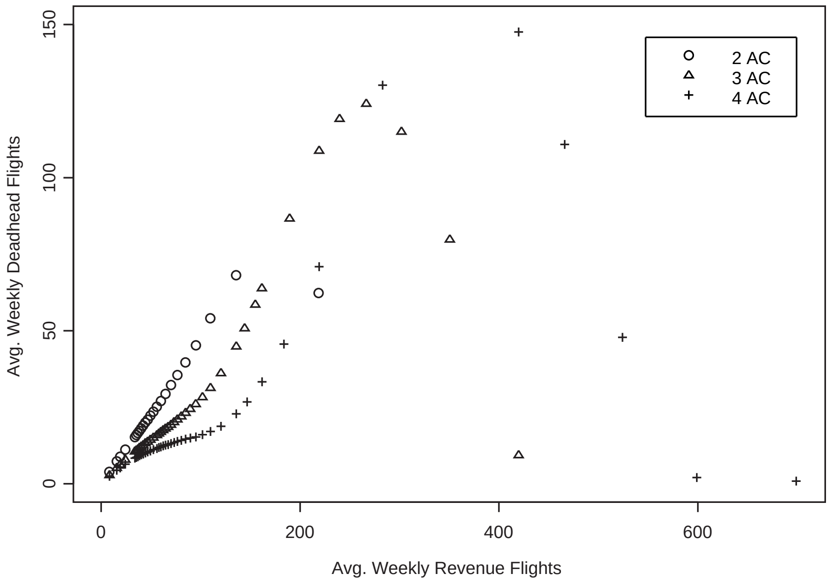

Figure 3: Scatter plot of weekly average deadhead flights versus weekly average revenue flights for several aircraft levels

where $A_{3_{ij}}$ and $A_{4_{ij}}$ are the parameters to be estimated for each route (i, j).

To fit (21) for each route (i, j) in an example scenario analogous to that in Section 2.3.1, we use simulation results (averaged over 1000 runs) for each route in the two aircraft case. Here, we perform a regression for each individual route, since the results from Section 2.3.1 indicate that the expected number of deadhead flights from the simulation do exhibit significant statistical differences between routes. Table 7 displays the parameter estimates for each route along with the associated standard errors, again using the nls routine in S-PLUS (Insightful 2004). Figure 4 displays a scatter of the average deadhead flights versus average revenue flights on each route with the corresponding fitted values from (21).

3.2.4 Penalty Cost

It remains to specify the weekly penalty cost for passenger service denials (i, j) H_{ij}, the cost incurred by the service provider when a certain type of passenger arrives over capacity for a flight and is subsequently denied service. We have previously described this cost as the

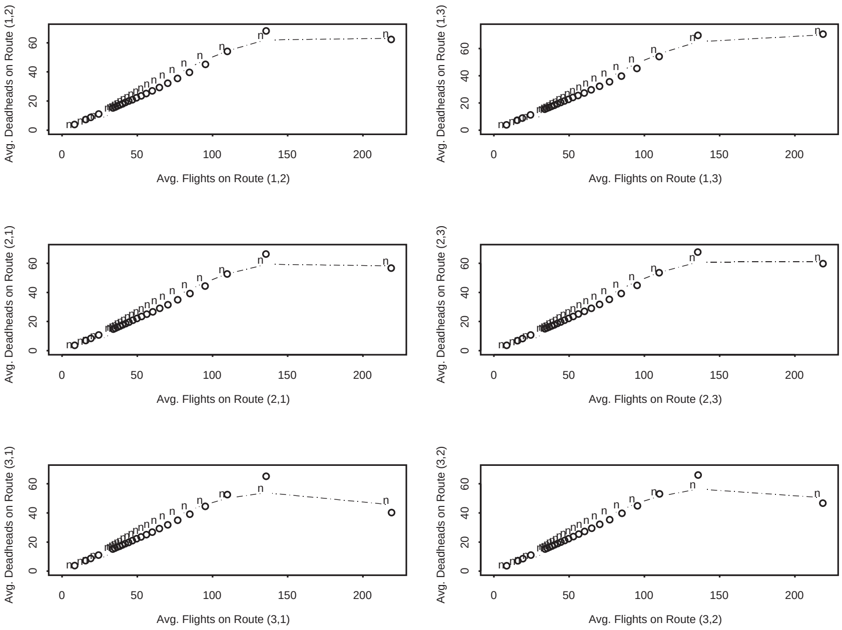

Figure 4: Scatter plots of weekly average deadhead flights versus weekly average revenue flights in two aircraft case with corresponding fitted values for (21) on each route (n=fitted values, o=observed values)

reimbursement of the current route fare to the denied passenger, plus a percentage credit towards a future flight. In general, $H_{ij}(P_{ij}) = R \cdot P_{ij}$ for all (i, j), where $1 \leq R < 2$. For our example scenario, we use $R = 1.1$.

4 Comparison and Validation

As described in Section 3, the flow model is an aggregate representation of the activity of an air taxi service network over the course of one week. One could avoid the potential complexity of a minute-by-minute representation of an air taxi service network (as in a discrete-event model like the one in Section 2.3) and instead use the flow model to determine average utilization behaviors of an air taxi service network over medium-range time horizons. In order to "validate" that the flow model acts "like" a lower-level model, we need to perform statistical tests on the equality of some of the key outputs for the two models. Since we are interested in the week-to-week basic utilization metrics of an air taxi service network, the key outputs of interest are the weekly expected number of passenger arrivals, revenue flights, and deadhead flights on each route, as well as the weekly average aircraft utilization (as defined in Section 2.3) for the entire network.

In Section 2.3.1, we have specified for our analysis a range of passenger arrival rates to each city in the service network. To "imply" the same arrival rates for the flow model, recall that the passenger arrivals to (i, j) during week n is given by (3). Assuming a homogeneous arrival process over the week, it follows that the hourly arrival rate to (i, j) is

$$\mu_{ij} = \frac{D_{ij}(P_{ij})}{168} \tag{22}$$

with $D_{ij}(P_{ij})$ is given in (3). Since it is assumed that customers arriving to each city then desire to go to each potential destination with equal probability, the arrival rates μ_{ij} to each (i, j) is just half the rate to city i. Thus, using (3) and (22), the weekly fare P_{ij} to be charged on (i, j) to imply the hourly arrival rate μ_i to city i is

$$P_{ij} = \frac{-\ln(168 \cdot 0.5 \cdot \frac{\mu_i}{B_{ij}})}{0.01} \tag{23}$$

giving the fares summarized in Table 8.

4.1 Results and Discussion

The values for passenger arrivals, revenue flights, and deadhead flights for one week using the flow model, as described in Section 3, are summarized in Table 9, 10, and 11. Likewise, the values for overall aircraft utilization for one week, using the flow model, are summarized in Table 12. In evaluating the validity of the flow model as a reasonable representation of the operations of an on-demand air taxi service network, the results in Tables 9, 10, 11, and 12 are treated as the "true" value X of weekly passenger arrivals, revenue flights, deadhead flights, or aircraft utilization under a particular hourly city-based arrival rate. We treat the analogous results in Tables 1, 2, and 3 as the sample mean $\overline{X}$ and sample standard deviation S, from a sample of n simulation runs, of weekly passenger arrivals, revenue flights, deadhead flights, or aircraft utilization under a particular hourly city-based arrival rate. We then test whether there is a significant difference between the outputs of the two models, where the flow model is treated as the "standard" air taxi service whose outputs are true parameter values and the discrete-event simulation produces outputs which are realizations of the "standard" service.

Table 13 summarizes the approximate p-values for the t-test ($df = 999$) applied to the weekly passenger arrivals on each route for each hourly city arrival rate and weekly deadhead flights on each route for a partial list of hourly city arrival rates. The results for the tests omitted from Table 11 show $p < 0.001$ for all routes. At a 0.05 level of significance, there is not a difference between the flow model results for weekly passenger arrivals and the average weekly passenger arrivals from the discrete-event model. However, the same is not true for the average weekly number of revenue flights, and only true for selected routes in low-demand cases for the average weekly number of deadhead flights.

Thus, we cannot claim to have "validated" that the flow model produces similar outputs to the discrete-event model. However, we can consider whether there is any practical significance to the statistical significance of the differences between the results of the flow model and those of the discrete-event model. In the case of the weekly revenue flights on each route, the flow model generally underestimates the discrete-event model by approximately one to three flights on each route, excluding the highest demand case. Similarly, for the weekly deadhead flights on each route, the differences between the results of the two models are approximately one to two flights (in either direction) on each route, excluding the highest demand cases. In a scenario with somewhere from 60 to 100 revenue flights and 30 to 50 deadhead flights, a difference of no more than two or three such flights might not affect how some providers would choose to set fares for each route since such decisions may be guided by other metrics, such as those related to flight costs and revenues. Similarly, the statistically significant differences in aircraft utilization between the two models is on the magnitude of less than one percent, even with a systematic disagreement between these numbers due to the disagreement between the flight and deadhead numbers. Furthermore, the larger differences in the highest demand cases can be discounted if we consider that it may be unrealistic to expect 2 to 3 potential passengers to show up each hour for each route.

5 Example Application of Flow Model: Pricing

The flow model makes it possible for a service provider to use nonlinear programming algorithms to compute fares on its routes. It is of generic interest for a service provider to determine the fares to charge on each route to optimize some objective function (e.g., maximizing profit), in conjunction with evolving network configuration and passenger demand conditions, and, in a potential extension, with respect to the actions of competing providers. A number of iterative methods exist for solving optimization problems over the course of a number of decision stages. One such class of methods, gradient-based methods (Bertsekas 1999), are often used to solve deterministic optimization problems (both constrained and unconstrained) with smooth objective functions, by using first-order information to make slight improvements in the objective function at each stage. The flow model, as presented in Section 3, thus lends itself to a pricing application in which a service provider can use a gradient-based method to iteratively determine a profit-maximizing set of fares on its service network.

5.1 Gradient Algorithm: Description and Convergence

The service provider's objective is to maximize its expected single-stage profit, as given in (15), subject to the constraint that the fare charged on each route is strictly positive, i.e., the provider faces the following problem:

$$\max_{P>0} E[Z(P)]. \tag{24}$$

P is the vector of decision variables, the fares charged on all routes in the provider's network. Although (24) is a constrained optimization problem, the set of feasible price vectors is open so we can still apply methods and convergence arguments for unconstrained problems.

To solve, we use the simple gradient ascent step (Bertsekas 1999) for each stage n

$$P^{n+1} = P^n + \alpha \cdot \nabla_{P^n} E[Z^n] \tag{25}$$

where α is a constant stepsize parameter, over all n, set to a sufficiently small value to ensure convergence of the steepest ascent algorithm and prevent the algorithm from moving to an infeasible point if it is close enough to the boundary of the feasible set (although it is intuitively clear that the algorithm would tend to move away from fares close to zero). Application of the steepest ascent step terminates after the first stage n where

$$\|\nabla_{P^n} E[Z^n(P^n)]\| \leq \epsilon_1 \tag{26}$$

where $\epsilon_1 > 0$ is a tolerance parameter set close to zero, to mimic the first-order necessary condition for a maximum that the gradient associated with the point equals zero.

The steepest ascent algorithm of (25) is known to converge to a local maximum for *unconstrained* problems, as long as (i) the objective function is twice continuously differentiable, (ii) the gradient of the objective function is Lipschitz continuous with some Lipschitz constant $L \geq 0$, and (iii) the constant stepsize parameter α is chosen such that $\epsilon_2 \leq \alpha \leq (2 - \epsilon_2)/L$, where $\epsilon_2 > 0$. (Bertsekas 1999) We verify the Lipschitz continuity of $\nabla_P E[Z(P)]$ below for $P > 0$. Thus, as long as α is also chosen sufficiently small to ensure all iterates of (25) are strictly positive, we may assert that (25) converges to a local maximum for the optimization model of (24).

To verify Lipschitz continuity, consider a network with N routes, let P_1 and P_2 be N-by-1 be arbitrary vectors of route fares. We denote any N-by-1 vector whose elements are $f(P_{ij})$ for any function f and every route (i, j) as simply $f(P)$. The gradient of the objective (expected weekly profit) function (15), taken with respect to the vector of fares P, is given by

$$\nabla_P E[Z(P)] = F_1(P) + F_2(P) + F_3(P) + F_4(P) + F_5(P) \tag{27}$$

where

$$F_1(P) \propto Pe^{-C_1 P} \tag{28}$$

$$F_2(P) \propto Pe^{-C_2 P} \tag{29}$$

$$F_3(P) \propto e^{-C_2 P} \tag{30}$$

$$F_4(P) \propto e^{-C_3 P - C_4 e^{-C_2 P}} \tag{31}$$

$$F_5(P) \propto Pe^{-C_5 P - C_6 e^{-C_7 P}} \tag{32}$$

where C_1, C_2, C_3, C_4, C_5, C_6, and C_7 are scalar constants, with only C_1 being necessarily positive. $F_1(P)$, $F_3(P)$, and $F_4(P)$ are, respectively, the gradients of the vector forms of (4), (12), and (11). $F_2(P)$ and $F_5(P)$ are the gradients of separate terms of the vector forms of (10).

Clearly, $F_1(P)$ is Lipschitz continuous. If $C_2 > 0$, then both $F_2(P)$ and $F_3(P)$ are also clearly Lipschitz continuous. Similarly if $C_3 P + C_4 e^{-C_2 P} > 0$ and $C_5 P + C_6 e^{-C_7 P} > 0$, then $F_4(P)$ and $F_5(P)$ are also clearly Lipschitz continuous, respectively. Note, however, that both $F_2(P)$ and $F_4(P)$, under the aforementioned conditions, are only Lipschitz continuous on the open feasible set defined in (24). Having established conditions for the Lipschitz continuity (on the open feasible set of strictly positive fares) of each term in (27), we can claim that (27) itself is also Lipschitz continuous on the open set of positive fares.

5.2 Results and Discussion

We now illustrate the application and convergence of the gradient algorithm in Section 5.1 using the simple air taxi service network scenario with three cities equidistant at 60 nm, and setting the parameters in the flow model as specified in Section 3.2. The initial vector of fares for the gradient algorithm is summarized in Table 14. Using these initial fares, the gradient ascent iteration in (25) proceeds with $\alpha_n = 0.005$ for each iteration n and terminates under the criterion (26) with $\epsilon = 10^{-6}$.

Implementing the model with MATLAB (Version 7.1) (Mathworks 2005), the terminal stage vectors of fares, demands, revenue flights, deadhead flights, and service denials are summarized in Table 15. These are the values for each set of variables at the end of the iteration when the termination criterion (26) is reached. The terminal stage expected profit is approximately \$21,727 and the terminal stage overall aircraft utilization is 0.3186.

Figures 5, 6, 7, 8, and 9 provide the iteration-by-iteration trajectory plots of the fare, demand, revenue flights, deadhead flights, and service denials, respectively, for each route. Figure 10 provides provides trajectory plots of the iteration-by-iteration expected profit and overall aircraft utilization. As the trajectory plots display, the service provider uses the gradient algorithm to increase the fare on each route, leading to associated decreases in the demand and flights (both revenue and deadhead) on each route. The provider transitions from sustaining large losses in early stages to significant profits in a matter of 70 iteration by controlling the fare so that passengers arrive to each of the route less frequently and thus, the flow of revenue flights eventually reaches a level of below 70 per iteration. As the gradient algorithm searches for a point maximizing the expected total single-iteration profit in this example, the service provider loses revenue (via passenger flow) but sacrifices these since a larger volume of passenger demand also results in higher single-iteration costs from deadheading and service denials, and thus higher single-iteration total costs.

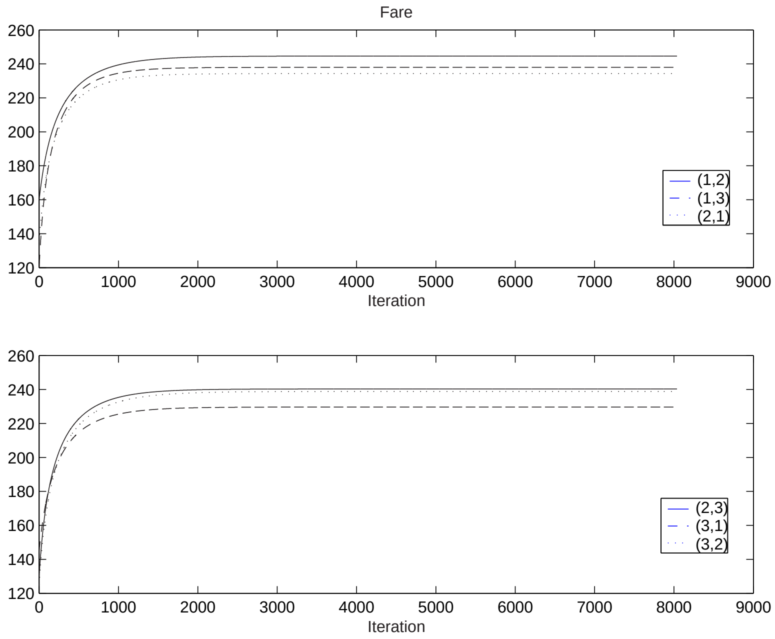

Figure 5: Iteration-by-iteration trajectories of fares on each route

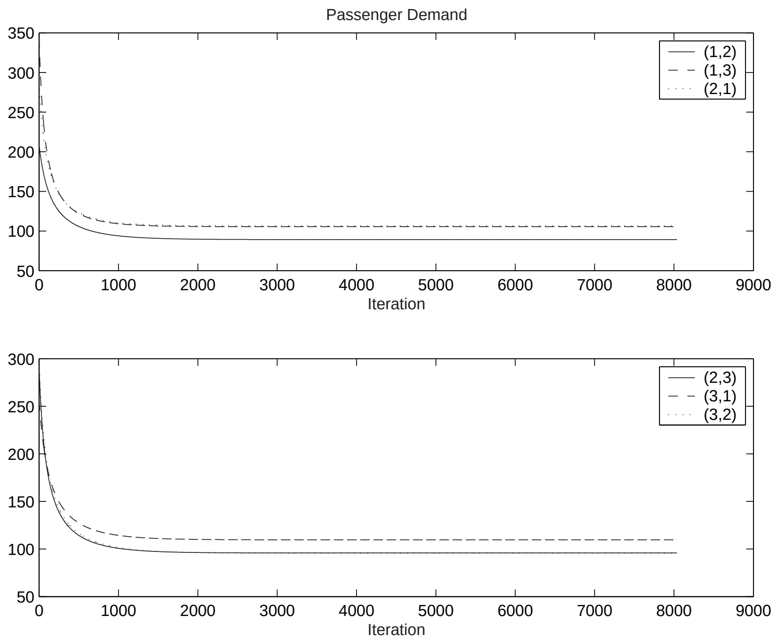

Figure 6: Iteration-by-iteration trajectories of passenger demands on each route

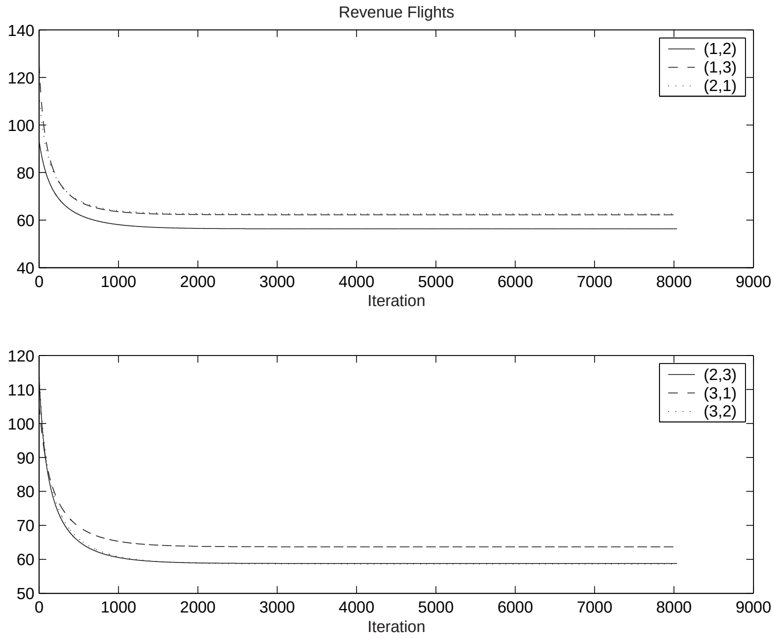

Figure 7: Iteration-by-iteration trajectories of revenue flights on each route

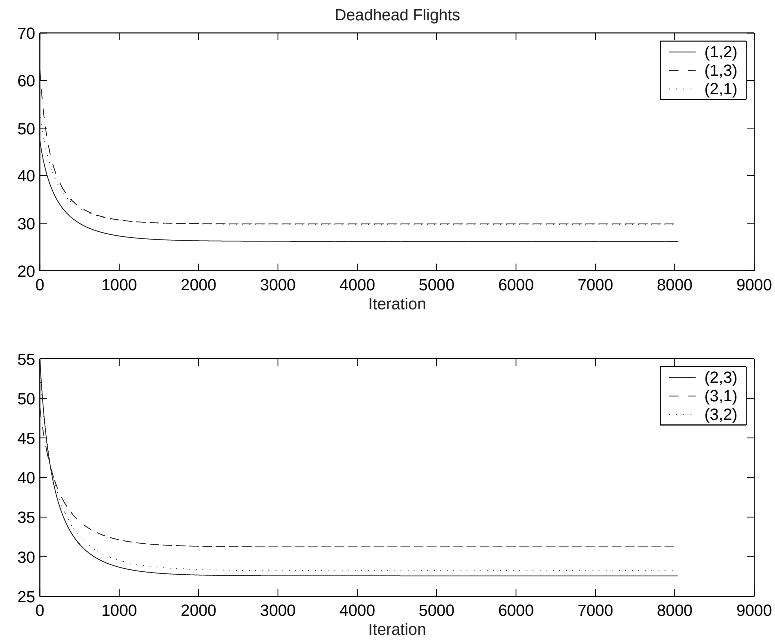

Figure 8: Iteration-by-iteration trajectories of deadhead flights on each route

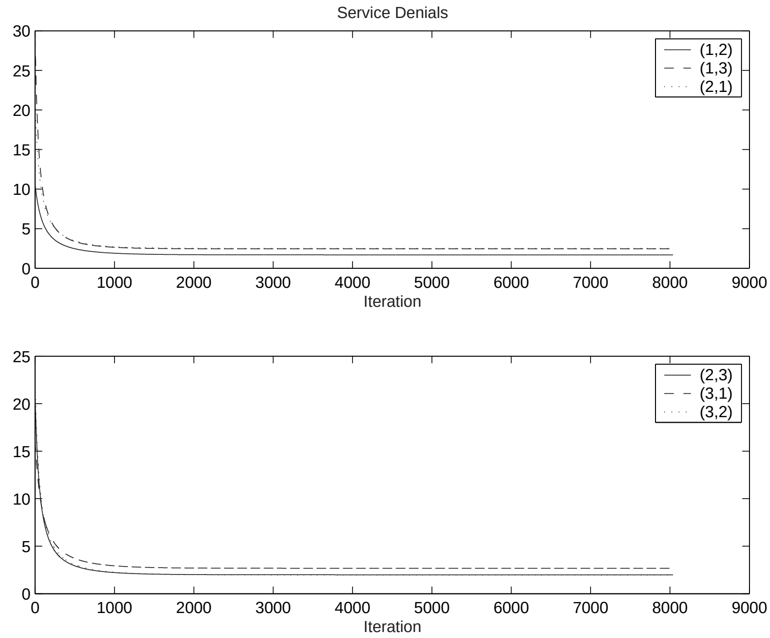

Figure 9: Iteration-by-iteration trajectories of service denials on each route

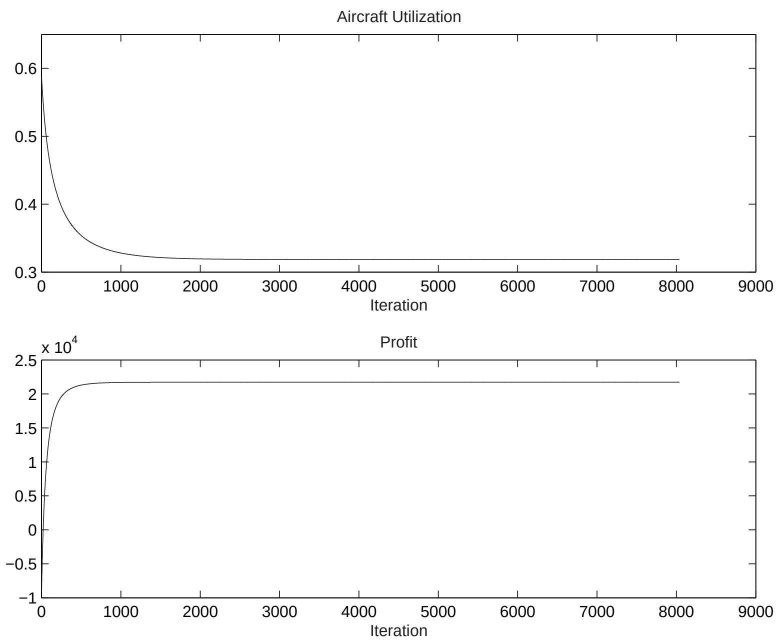

Figure 10: Iteration-by-iteration trajectories of the total single-stage profit and aircraft utilization along the entire network

6 Conclusion and Future Paths

The main contributions of this paper are that (i) we propose a general service model to guide both the modeling of on-demand air taxi services for a single provider and a discrete-event representation for a potential air taxi service, (ii) we formulate an extended flow model (based on Lee et al. 2005) that captures the discrete-event model at an aggregate level, and (iii) we illustrate the application of the flow model to service pricing using gradient-based optimization methods. Preliminary results demonstrate that the flow model produces some outputs which are not statistically different from those produced by the discrete-event model. In addition, we have observed that gradient-based pricing converges to a stationary set of fares.

The air taxi service model presented here is a first attempt to prescribe a general structure for air taxi service modeling, focusing only on the basic operation for such a service. In the future, we wish to expand the current service model to incorporate the effects of weather and aircraft maintenance or to include time-of-day variation in the passenger arrival rate or the availability of service. The service model may also be extended to a case with multiple service providers to study how competition affects pricing decisions for providers and demand from arriving passengers.

A Parameters and Results for the Numerical Example of §2.3.1

A.1 Discrete-Event Model Results

Passenger Arrivals							
		(i, j)					
μ_i		(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	Mean	83.87	84.07	83.87	84.34	83.92	83.93
	S.D.	9.55	9.21	9.55	9.32	8.99	9.29
1.11	Mean	93.19	93.47	93.19	93.80	93.16	93.29
	S.D.	10.03	9.49	10.03	9.68	9.65	9.83
1.25	Mean	104.78	104.90	104.78	105.52	104.67	105.04
	S.D.	10.70	9.91	10.70	10.28	10.20	10.32
1.43	Mean	119.87	119.91	119.87	120.49	119.47	119.89
	S.D.	11.40	10.74	11.40	11.06	10.98	10.95
1.67	Mean	139.78	140.01	139.78	140.12	139.52	139.67
	S.D.	11.99	11.61	11.99	11.91	11.93	11.87
2.00	Mean	167.73	167.96	167.73	167.89	167.51	167.88
	S.D.	12.98	13.06	12.98	12.92	13.33	13.09
2.50	Mean	209.67	210.38	209.67	209.54	209.58	209.82
	S.D.	14.65	14.41	14.65	14.76	14.93	14.72
3.33	Mean	279.56	280.12	279.56	279.07	279.30	279.54
	S.D.	17.45	16.96	17.45	16.86	16.81	16.20
5.00	Mean	419.02	419.46	419.02	418.90	419.22	419.47
	S.D.	21.65	21.05	21.65	20.60	20.63	19.88

Table 1: Weekly averages and standard deviations over 1000 simulation runs for passenger arrivals for all routes and hourly arrival rates

Revenue Flights							
		(i, j)					
μ_i		(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	Mean	56.15	56.16	56.24	56.13	56.06	55.93
	S.D.	5.25	5.14	5.00	4.99	5.01	5.15
1.11	Mean	60.12	60.20	60.33	60.15	60.11	60.03
	S.D.	5.13	5.02	5.05	5.05	5.02	4.99
1.25	Mean	64.76	64.73	65.03	64.86	64.65	64.77
	S.D.	5.11	4.98	5.05	4.98	4.94	4.91
1.43	Mean	70.19	70.26	70.43	70.31	70.06	70.28
	S.D.	4.99	4.99	5.08	4.95	4.90	4.89
1.67	Mean	76.74	76.76	76.83	76.91	76.61	76.79
	S.D.	4.63	4.881	5.00	4.75	4.76	4.76
2.00	Mean	84.81	84.77	84.84	85.01	84.74	84.94
	S.D.	4.55	4.77	4.75	4.62	4.70	4.63
2.50	Mean	95.15	95.39	95.25	95.44	95.23	95.36
	S.D.	4.46	4.66	4.53	4.50	4.49	4.44
3.33	Mean	109.92	110.06	109.76	109.90	109.89	110.01
	S.D.	4.49	4.41	4.20	4.18	4.26	4.23
5.00	Mean	135.73	135.86	135.75	135.85	135.71	135.71
	S.D.	4.54	4.46	4.42	4.40	4.48	4.26

Table 2: Weekly averages and standard deviations over 1000 simulation runs for revenue flights for all routes and hourly arrival rates

Deadhead Flights							
		(i, j)					
μ_i		(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	Mean	25.33	25.21	25.20	24.79	25.05	24.94
	S.D.	4.43	4.56	4.44	4.60	4.47	4.49
1.11	Mean	27.23	27.04	27.11	26.70	26.94	26.94
	S.D.	4.60	4.53	4.68	4.71	4.65	4.55
1.25	Mean	29.41	29.39	29.20	28.90	29.09	29.20
	S.D.	4.70	4.58	4.78	4.76	4.66	4.84
1.43	Mean	31.97	32.24	31.86	31.59	31.73	31.80
	S.D.	4.65	4.83	5.03	4.90	4.70	4.93
1.67	Mean	35.25	35.52	34.91	34.81	34.87	34.90
	S.D.	4.92	4.94	5.09	4.84	4.90	5.02
2.00	Mean	39.74	39.78	38.95	39.22	39.11	39.06
	S.D.	5.20	5.10	5.29	5.03	5.17	4.94
2.50	Mean	45.21	45.52	44.52	44.91	44.63	44.41
	S.D.	5.58	5.63	5.45	5.15	5.57	5.35
3.33	Mean	53.95	54.09	52.51	53.06	52.31	52.72
	S.D.	6.02	5.73	5.89	5.66	5.73	5.70
5.00	Mean	68.86	69.47	66.63	67.62	65.18	65.66
	S.D.	6.37	6.46	6.29	6.29	6.43	6.40

Table 3: Weekly averages and standard deviations over 1000 simulation runs for deadhead flights for all routes and hourly arrival rates

μ_i	1.00	1.11	1.25	1.43	1.67	2.00	2.50	3.33	5.00
Mean	0.2900	0.3112	0.3357	0.3647	0.3993	0.4434	0.5006	0.5822	0.7250
S.D.	0.0117	0.0119	0.0119	0.0123	0.0122	0.0126	0.0131	0.0132	0.0133

Table 4: Weekly averages and standard deviations over 1000 simulation runs for overall aircraft utilization for all hourly arrival rates

	Passenger Arrivals		Revenue Flights		Deadhead Flights	
μ_i	F	p-value	F	p-value	F	p-value
1.00	0.3402	0.8887	0.4443	0.8176	1.9443	0.0837
1.11	0.6192	0.6852	0.4083	0.8434	1.5166	0.1809
1.25	0.8450	0.5176	0.6912	0.6301	1.6382	0.1462
1.43	0.8613	0.5063	0.6321	0.6752	2.1783	0.0537
1.67	0.4412	0.8199	0.4142	0.8392	3.2086	0.0068
2.00	0.2169	0.9555	0.4962	0.7794	4.9311	< 0.001
2.50	0.4355	0.8240	0.5934	0.7051	6.2489	< 0.001
3.33	0.4580	0.8077	0.6013	0.6990	16.8097	< 0.001
5.00	0.1454	0.9814	0.2606	0.9346	73.3989	< 0.001

Table 5: F statistic and p-values for ANOVA testing equality of average passenger arrivals, and revenue and deadhead flights across all routes for example in Section 2.3.1

A.2 Flow Model Identification and Validation Results

(i,j)	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
B_{ij}	1030	1140	1010	1060	1090	1050

Table 6: Maximum Demand Parameter Values for Validation and Pricing Scenarios

(i,j)	$A_{3_{ij}}$ (t)	$A_{4_{ij}}$ (t)	S.E.
(1,2)	0.0156 (24.021)	0.0113 (33.785)	2.8914
(1,3)	0.0149 (23.133)	0.0106 (31.732)	3.0767
(2,1)	0.0158 (24.453)	0.0117 (34.811)	2.7602
(2,3)	0.0157 (24.647)	0.0115 (34.836)	2.7813
(3,1)	0.0182 (18.580)	0.0135 (27.699)	3.4807
(3,2)	0.0174 (21.447)	0.0128 (31.447)	3.0933

Table 7: Parameter estimates (and associated t-statistics) and standard errors for (21) for each route in example scenario

μ_i	P_{12}	P_{13}	P_{21}	P_{23}	P_{31}	P_{32}
1.00	250.6497	260.7967	258.1298	253.5207	256.3116	251.6159
1.11	240.2137	250.3607	247.6938	243.0847	245.8756	241.1799
1.25	228.3354	238.4823	235.8155	231.2064	233.9973	229.3016
1.43	214.8823	225.0292	222.3624	217.7533	220.5442	215.8485
1.67	199.3674	209.5143	206.8475	202.2384	205.0293	200.3336
2.00	181.3350	191.4820	188.8151	184.2060	186.9969	182.3012
2.50	159.0207	169.1676	166.5008	161.8917	164.6825	159.9868
3.33	130.3525	140.4994	137.8326	133.2235	136.0144	131.3187
5.00	89.7059	99.8529	97.1861	92.5769	95.3678	90.6721

Table 8: Route fares on each route in flow model implying hourly arrival rate μ_i to origin city i

Passenger Arrivals						
	(i,j)					
μ_i	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	84.00	84.00	84.00	84.00	84.00	84.00
1.11	93.24	93.24	93.24	93.24	93.24	93.24
1.25	105.00	105.00	105.00	105.00	105.00	105.00
1.43	120.12	120.12	120.12	120.12	120.12	120.12
1.67	140.28	140.28	140.28	140.28	140.28	140.28
2.00	168.00	168.00	168.00	168.00	168.00	168.00
2.50	210.00	210.00	210.00	210.00	210.00	210.00
3.33	279.72	279.72	279.72	279.72	279.72	279.72
5.00	420.00	420.00	420.00	420.00	420.00	420.00

Table 9: Weekly passenger arrivals from the flow model for all routes and arrival rates

Revenue Flights						
	(i, j)					
μ_i	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	54.33	54.33	54.33	54.33	54.33	54.33
1.11	57.82	57.82	57.82	57.82	57.82	57.82
1.25	62.06	62.06	62.06	62.06	62.06	62.06
1.43	67.24	67.24	67.24	67.24	67.24	67.24
1.67	73.76	73.76	73.76	73.76	73.76	73.76
2.00	82.14	82.14	82.14	82.14	82.14	82.14
2.50	93.83	93.83	93.83	93.83	93.83	93.83
3.33	111.33	111.33	111.33	111.33	111.33	111.33
5.00	141.87	141.87	141.87	141.87	141.87	141.87

Table 10: Weekly numbers of revenue flights from the flow model for all routes and arrival rates

Deadhead Flights						
	(i,j)					
μ_i	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	24.92	24.72	24.70	24.81	25.80	25.62
1.11	27.13	26.98	26.85	26.99	27.87	27.75
1.25	29.80	29.72	29.44	29.62	30.33	30.28
1.43	32.99	33.03	32.53	32.76	33.20	33.27
1.67	36.88	37.09	36.27	36.57	36.58	36.83
2.00	41.60	42.09	40.77	41.19	40.51	41.02
2.50	47.57	48.52	46.41	46.99	45.15	46.09
3.33	54.95	56.74	53.23	54.09	50.18	51.87
5.00	63.19	66.66	60.47	61.82	53.96	56.97

Table 11: Weekly numbers of deadhead flights from the flow model on all routes and all hourly arrival rates

μ_i	1.00	1.11	1.25	1.33	1.67	2.00	2.50	3.33	5.00
U	0.2836	0.3039	0.3283	0.3579	0.3945	0.4405	0.5022	0.5887	0.7227

Table 12: Weekly aircraft utilization of flow model for all hourly arrival rates

Passenger Arrivals						
	(i, j)					
μ_i	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
1.00	0.6669	0.8101	0.6669	0.2489	0.7785	0.8117
1.11	0.8748	0.4436	0.8748	0.0676	0.7933	0.8722
1.25	0.5157	0.7497	0.5157	0.1100	0.3065	0.9025
1.43	0.4882	0.5365	0.4882	0.2904	0.0615	0.5067
1.67	0.1876	0.4766	0.1876	0.6711	0.0442	0.1045
2.00	0.5108	0.9229	0.5108	0.7878	0.2453	0.7720
2.50	0.4764	0.4045	0.4764	0.3246	0.3739	0.6991
3.33	0.7719	0.4560	0.7719	0.2231	0.4297	0.7254
5.00	0.1526	0.4174	0.1526	0.0916	0.2321	0.3994
Deadhead Flights						
1.00	0.0035	< 0.001	< 0.001	0.8907	< 0.001	< 0.001
1.11	0.4920	0.6754	0.0793	0.0518	< 0.001	< 0.001
1.25	0.0088	0.0229	0.1127	< 0.001	< 0.001	< 0.001
2.50	< 0.001	< 0.001	< 0.001	< 0.001	0.0032	< 0.001

Table 13: Approximate p-values from t-tests ($df = 999$) for the equality of weekly passenger arrivals and deadhead flights between the flow model and discrete-event model

A.3 Parameters and Results from Pricing Application

(i,j)	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
P_{ij}^1	160	120	135	130	145	125

Table 14: Initial Route Fares in Gradient Pricing Algorithm

(i,j)	(1,2)	(1,3)	(2,1)	(2,3)	(3,1)	(3,2)
P_{ij}	244.60	237.96	234.28	240.32	229.67	238.83
D_{ij}	89.24	105.55	106.63	95.86	109.65	95.45
S_{ij}	56.32	62.25	62.63	58.78	63.68	58.63
DH_{ij}	26.19	29.85	29.79	27.59	31.24	28.24
O_{ij}	1.699	2.462	2.517	1.991	2.676	1.972

Table 15: Terminal Stage Values for Variables

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